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ABSTRACT

This report of the fourth British Columbia Mathematics Assessment conducted in May 1990 is a condensed form of the technical report of the evaluation project. The report is divided into five sections. An introduction provides the rationale and purpose of the project and describes the organization and scope of the assessment. The second section describes the findings about students' and teachers' opinions concerning issues related to the teaching and learning of mathematics. The third section discusses students' achievement in mathematics on a topic-by-topic basis. It describes students' performance by establishing four achievement categories for each grade level according to the percentage of students that are able to perform those mathematical skills. Mathematical skills and knowledge for each of the categories at the different grade levels are described. The fourth section consists of a more detailed discussion of students' achievement according the following five strands: number and operations, data analysis, geometry, measurement, and algebra. The final section presents conclusions and identifies four areas in which improvement is needed. The areas are: participation of women, implementation of the curriculum, teaching practices employed, and student outcomes. A "Highlights" section appears at the beginning of the document that identifies the questions addressed by the evaluation and a summary of the findings concerning students, teachers, students' achievement, and students' achievement on particular topics. Two appendices provide a description of the authors and several tables listing Review and Interpretation Panel members. (MDH)

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Mathematics '90: A Status Report on School Mathematics in British Columbia

Provincial Report
1990

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Mathematics '90: A Status Report on School Mathematics in British Columbia

Provincial Report

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PREFACE

This *Provincial Report* of the 1990 British Columbia Mathematics Assessment includes a description of each of the major components of the study, an analysis of the major findings, and several suggestions for improvement directed at policy makers in the educational system of the province. The goals of the assessment were to monitor patterns of students' achievement in and attitudes toward mathematics, to trace changes in that achievement since the 1985 assessment, to obtain information about the degree to which the revised curriculum has been implemented, and to conduct a survey of instructional practices employed by teachers of mathematics.

The members of the Contract Team who carried out this assessment on behalf of the Ministry of Education are grateful to all of the teachers, department heads, principals, and school district personnel throughout the province for their cooperation. In addition, special thanks are due to Sandra Crespo, Thomas García, Mike Marshall, and the staff of the Education Measurement Research Group at U.B.C. whose assistance in the analysis of the data from the assessment and in the preparation of this report was invaluable.

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Highlights

Every large-scale evaluation project produces large numbers of findings, and this fourth mathematics assessment is no exception. The *Technical Report* of the assessment includes a comprehensive discussion of our findings, as well as a great deal of background and explanatory material. This *Provincial Report* discusses many of those same findings, but in a more condensed form. In this section, some of the more important and more interesting findings are presented in point form. This report contains no formal recommendations. Instead, suggestions for improvement are included throughout the report, and these are addressed to all of the major stakeholders in mathematics education in the province. Readers interested in more information about a particular topic or in a more complete discussion of the implications of the findings should refer to the relevant sections of the *Technical Report,* or to later sections of this report.

The fourth mathematics assessment was carried out in all of the schools of the province in May, 1990. All students enrolled in Grades 4, 7, and 10 completed test booklets developed for the study, and their teachers completed questionnaires dealing with a variety of aspects of the teaching and learning of mathematics. The major questions addressed in the assessment were the following:

- To what extent are students in Grades 4, 7, and 10, in both the public and independent school systems of the province, achieving the objectives of the prescribed mathematics curriculum?

- What mathematical concepts, skills, and techniques have students at each of the grade levels mastered? That is to say, what mathematics can these students do?

- To what extent have levels of achievement changed from those described in the reports of the mathematics assessments conducted in 1977, 1981, and 1985?

- To what extent are topics such as data analysis and transformational geometry, which are either new to the curriculum or have been accorded increased prominence in the new curriculum, being taught in the classrooms of the province?

- What kinds of teaching practices do teachers of mathematics employ, and how have those teaching practices been affected by the introduction of the revised curriculum?

- What are teachers' and students' opinions about how important selected curricular topics are, how easy those topics are to learn or to teach, how much they enjoy learning or teaching those topics, and to what extent do their opinions differ?

The results of the study indicate that the students in our schools are learning a great deal of the mathematics prescribed for them in the curriculum, and there is also evidence that their achievement has improved over time. An-

other major finding is that teachers generally support the new directions established in the revised curriculum, and the effects of the implementation of that curriculum can be seen throughout the province.

There are a number of areas where the members of the Contract Team believe that improvements can and should be made. Some of these concern parts of the curriculum that require additional attention. Others concern teachers, their academic and professional preparation, and the teaching practices they employ. A third set concerns matters beyond the level of the individual classroom: that is, schools, school districts, and the province as a whole.

The following are the highlights of the assessment. In a number of cases, some sort of remedial action seems to us to be required and, in those instances, we have made specific suggestions. The last section of this report contains several other suggestions that the Contract Team wishes to address to major stakeholders in the province.

Students

- Students' plans for the future are very positive. More than half of Grade 10 students expect to continue studying mathematics through Grade 12, and the majority also plan to pursue their education at the post-secondary level.
- The alternative courses—Mathematics 9A and 10A, and Introductory Mathematics 11—are seen by students as viable routes to Mathematics 11. About one quarter of Grade 10 students are enrolled in the new Mathematics 10A, and the remainder in Mathematics 10.
- Students believe that mathematics is needed for and used in many desirable occupations, yet their interest in the subject declines as they continue to study it. Only a slight majority of Grade 4 students, and a minority of those in Grades 7 and 10, say that they would like to have jobs involving the use of mathematics.
- Learning strategies for problem solving was identified by students as the most difficult topic in the curriculum. Their interest in and enjoyment of problem solving declines with grade level.
- Students generally assign the highest degree of importance to traditional topics such as computational skills, familiar topics, and those with obvious applications.
- Students at all grade levels report that the teaching of mathematics is highly traditional in character. Even at the Grade 4 level, students report that they seldom make use of concrete materials or work together in small groups: two practices that research has shown to be highly effective in promoting students' learning, and that are hallmarks of the revised curriculum.
- Differences in achievement between males and females have continued to decrease, and almost all such differences are now so small as to be of little, if any, educational significance.
- There is evidence of a steady improvement in students' performance in mathematics over the past 15 years. Almost all such changes are relatively small, on the order of five percentage points or less; however, consistent

changes of that magnitude at the level of the entire school system are significant and encouraging.

- There has been a dramatic increase since 1985 in the extent to which calculators are used in mathematics classes, and now over 80 percent of Grade 10 students report that they use calculators frequently in their mathematics classes. However, despite strong and clear curricular support for the use of calculators as instructional aids and as computational aids in Grade 4 and Grade 7, about half the students in these grades report that they rarely or never use calculators in mathematics classes. Computer usage in mathematics classes is low; but, given the current state of the educational software available and the content of the curriculum, this is not necessarily a negative finding. That is to say, the content of the mathematics curriculum through Grade 10 may be such that the mathematics software which is currently available is either not suitable or not particularly helpful.

Teachers

- There has been a substantial influx of new teachers into the system since the last mathematics assessment in 1985.
- Less than 20 percent of Grade 10 mathematics teachers are women, and less than 20 percent of Grade 4 mathematics teachers are male. These imbalances seem not to have changed substantially since previous assessments, and this is an area of considerable concern.
- While the overall picture regarding the academic and professional backgrounds of teachers of mathematics in the province is fairly good, we do have two concerns. First, a large proportion of teachers of Mathematics 10A have little or no academic preparation in mathematics or in the teaching of mathematics. Previous assessment reports have raised this same issue, but the situation seems not to have improved at all. School- and district-level administrators need to become more sensitive to this matter, and take steps to ensure that mathematics classes are taught by teachers with appropriate qualifications. Second, the results show that the proportion of teachers of mathematics with the highest qualifications is decreasing. In other words, the numbers of teachers who are being asked to teach mathematics and who are not fully qualified to do so are increasing.
- Teachers seem to be spending less time teaching mathematics than they did in 1985. More than half report that they spend less than 45 minutes per day on mathematics on the average. This is a trend that could have serious implications for the mathematics curriculum, and schools should take steps to reverse it.
- Teachers express general approval of the revised curriculum that was introduced in 1987, for the curriculum guides produced by the Ministry of Education, and for the textbooks which have been approved for use in the schools. They also indicate that their approaches to teaching and evaluation have changed as a result of the implementation of the new curriculum.
- Teachers' support in principle for the revised curriculum is not always realized in practice. The assessment results show that teachers are not teaching some topics in the revised curriculum, and the topics which they are omit-

ting are among those that are new to the curriculum. For example, many teachers at all grade levels say that they have not taught topics in data analysis and geometry, two areas which have been given increased prominence in the revised curriculum.

- Increased emphasis on the development of students' skills in problem solving is one of the hallmarks of the revised curriculum, and teachers indicate that they are giving that topic increased attention. They also report that problem solving is the most difficult topic to teach, and their students agree.

What Mathematics Can Students Do?

- At each grade level, four categories or levels of student achievement were defined by the interpretation panels set up for the assessment to describe the kinds of mathematical competencies displayed by students. Category 1, the lowest level, is a kind of minimal competency level: a level of performance that might be expected of virtually every student. Category 4, on the other hand, includes only those students who have reached a relatively sophisticated level of performance in mathematics, and few students would be expected to reach this level.
- Over 95 percent of students in Grade 4 and Grade 7, and a similar proportion of those studying Mathematics 10, satisfied the requirements for Category 1. That is to say, almost all of those students satisfied the minimum requirements established by the interpretation panels.
- Less than 10 percent of students satisfied the Category 4 requirements for their grade. This is not surprising given the nature of those requirements.
- The achievement levels of students in Mathematics 10A are cause for concern. Over 20 percent of these students failed to satisfy even the Category 1 requirements for that program, and their performance on almost all items was quite weak. This state of affairs must be improved. Teachers and teacher educators need to identify better means for reaching these students and improving their performance.
- There is considerable overlap in the achievement distributions of the Mathematics 10 and Mathematics 10A students. That is to say, some Mathematics 10A students performed better on items that both groups of students wrote than many Mathematics 10 students. This could mean that the processes by which students are counselled or directed into Mathematics 10A, and other courses of this kind, are flawed. Principals, teachers, and counsellors should ensure that the methods employed to assist students and their parents in making course selection decisions are educationally valid and equitably applied.

Students' achievement on particular topics

- Students at all three grade levels generally performed better on fraction and decimal items involving pictorial representations or real-life situations than on items involving isolated computations. This suggests that students' understanding of rational number concepts might be enhanced through work with concrete representations, such as manipulative materials, and with opportunities to experience the development of concepts in situations close to

their personal experiences.

- Grade 4 and Grade 7 students had difficulty estimating quantities. Almost all Grade 7 teachers indicated that the mathematics required to answer such items had been taught, but students' performance was still not strong. More attention needs to be given to this important topic, and research studies should be done to identify appropriate teaching strategies.
- Data analysis is one of the topics that has been given increased emphasis in the revised curriculum, and it is encouraging to find that many teachers have begun to introduce some data analysis concepts to their students. Students at all grade levels were generally able to collect and interpret data in graphs and tables in order to determine single values or detect relationships. However, students did poorly on items involving elementary ideas of probability. Less than half the teachers indicated that they had taught this content, and this undoubtedly accounts for the relatively poor performance. Teachers must find the time to treat these important topics.
- Students' performance on problem-solving items was weaker than expected, and there is evidence that they had not had much experience in the use of general strategies for problem solving. This topic is a major thrust of the revised curriculum, and teachers need to give it increased attention.

Introduction

Mathematics has always been an essential component of the school curriculum. Plato included mathematics in the curriculum for the education of the philosopher-king in *The Republic,* and the subject has traditionally occupied a major place in the curriculum of every country. If anything, the importance of mathematics as a component of general education is increasing today, as governments come to appreciate the importance of mathematics and science education to the health of their economies and to the future success and viability of their countries.

In British Columbia, mathematics occupies a place second only to that of language in the curriculum. All students in the province study mathematics in each of their first eleven years of schooling, and a large percent of students also study mathematics in Grade 12. A recently revised mathematics curriculum (British Columbia Ministry of Education, 1987, 1988) has been in place in the schools of the province for the past three years, and that curriculum reflects many of the trends and recommendations which are influencing mathematics curricula internationally.

The fourth British Columbia Mathematics Assessment was conducted in May of 1990. Its purpose was to evaluate the state of mathematics teaching and learning in the schools. All students enrolled in Grades 4, 7, and 10 were expected to participate, as were their teachers. Students provided data about their achievement, their attitudes toward mathematics, their backgrounds and future plans, and their opinions about a variety of matters related to their school experience. Teachers were asked to provide information about their academic and professional backgrounds, about the kinds of teaching practices they employed in their teaching of mathematics, and about the extent to which they were teaching the topics prescribed in the revised curriculum guides for mathematics.

This mathematics assessment, like each of its predecessors, was an enormous undertaking, involving over 100 000 students throughout the province and their teachers. Our goal in writing this report has been to extract from the massive amounts of data collected under the auspices of the assessment those findings which most clearly illuminate the state of mathematics education in the province today. In so doing, many of the details have had to be excluded because of limitations of space. Readers interested in a more detailed presentation of the results and findings should consult the *Technical Report* of the assessment (Robitaille, 1991).

Organization of the Assessment

As with any other large-scale evaluation project, several groups of people representing the major stakeholders in mathematics education in the province participated in planning and carrying out the 1990 Mathematics Assessment. These included the members of the Contract Team which had the overall responsibility for carrying out the assessment, the staff of the Assessment, Examinations, and Reporting Branch of the Ministry of Education, the members of the Advisory Committee, the Review Panels, the Marking Panels, and the Interpretation Panels.

The Contract Team was responsible for designing the assessment, developing the instruments to be used in the project, and writing the final reports. The team included two members of the Department of Mathematics and Science Education at the University of British Columbia, a secondary school principal from the Langley School District, the Director of Instruction from the Coquitlam School District, and a curriculum consultant from the Vancouver School Board. They were assisted by a teacher, a vice-principal, and a consultant from Richmond, and a department head from North Vancouver. The names of the members of the Contract Team and of each of the other major committees set up for the 1990 assessment are listed in the *Technical Report* and at the beginning and end of this report.

The Advisory Committee provided guidance and advice to the Contract Team on the development of the instruments and a variety of other matters concerning the study. They were selected from across the province by the Assessment, Examinations, and Reporting Branch to represent a cross-section of opinion on matters related to the teaching and learning of mathematics. The committee included among its members teachers, school- and district-level administrators, and two university professors. The chairperson of the Contract Team and two members of the Assessment, Examinations, and Reporting Branch served as *ex officio* members of the Advisory Committee.

The Review Panels, the Marking Panels, and the Interpretation Panels were made up of educators and informed members of the public selected to represent all areas of the province and all levels of education in the K-12 system. The task of the Review Panels was to discuss the objectives of the assessment, and to comment on the test items, attitude scales, and questionnaires developed for use in the project. It was the responsibility of the Interpretation Panels to consider the results of the assessment and to make recommendations about students' performance on the tasks.

Scope of the Assessment

Systematic collection of reliable data about schools and the knowledge, skills, and attitudes of students is an essential component of the responsible management of education in the province. So too is the dissemination of the findings arrived at through the analysis of the data. Decisions about new directions or new emphases in the educational system should be based on a thorough understanding of what students have learned and how they are being taught. The purpose of the Learning Assessment Program in British Columbia is precisely that: to collect information about what students are being taught and what they are learning, and to make that information available to everyone who has an interest in the outcomes of education in this province. This includes not only parents and teachers, but also taxpayers, employers, educators at all levels, policy makers, administrators, researchers, and elected officials.

Within the framework of these general guidelines, the 1990 Mathematics Assessment was designed to address the following major questions:

- To what extent are students in Grades 4, 7, and 10, in both the public and the independent school systems of the province, achieving the objectives of the prescribed mathematics curriculum?

- What mathematical concepts, skills, and techniques have students at each of the grade levels mastered? That is to say, what mathematics can these students do?
- To what extent have levels of achievement changed from those described in reports of the mathematics assessments conducted in 1977, 1981, and 1985?
- To what extent are topics such as data analysis and transformational geometry, which are either new to the curriculum or have been accorded increased prominence in the new curriculum, being taught in the classrooms of the province?
- What kinds of teaching practices do teachers of mathematics employ, and how have those teaching practices been affected by the introduction of the new curriculum?
- How do teachers' and students' opinions differ concerning how important selected curricular topics are, how easy those topics are to learn or to teach, and how much do they enjoy learning or teaching those topics?

At the Grade 4 and Grade 7 levels there is a single mathematics curriculum prescribed by the Ministry of Education, and it is expected that all teachers will follow that curriculum using one of the textbooks approved for that grade.

For Grade 10 two courses have been prescribed: Mathematics 10 and Mathematics 10A. Mathematics 10 is part of a sequence of courses leading directly to Mathematics 11, which is required for entry to most post-secondary institutions. Mathematics 10A is part of a sequence of courses intended for students who have experienced difficulty in mathematics. Students who complete Mathematics 10A may take Introductory Mathematics 11 followed by Mathematics 11, or they may meet the mathematics graduation requirement by taking either Introductory Mathematics 11 or Mathematics 11A. The revised structure of courses in Grades 9, 10, and 11 gives all students a route into Mathematics 11, while providing alternatives for those students requiring a slower pace of instruction and a delayed introduction of the more abstract content.

The contents of the two Grade 10 mathematics courses are significantly different from one another, and it would have made little sense to ask all students to respond to the same achievement items. On the other hand, there is some overlap between the two, so it was not necessary to develop two completely different sets of items. The Grade 10 booklets were therefore divided into three sections–X, Y, and Z–with 20 items in each, and each student responded to two of the three parts. Students who were enrolled in Mathematics 10 responded to Parts X and Z, while students in Mathematics 10A responded to Parts X and Y.

Achievement Measures

The primary objective of every provincial assessment is to obtain valid information about students' achievement in particular areas of the cur-

riculum. The focus of attention in this report is the educational system as a whole, and the concern is for how well that system is performing rather than each student, each school, or each school district.

For the achievement monitoring component of the assessment, a two-way table of specifications was developed, with mathematical content along one dimension and cognitive behavior levels along the other. Like the curriculum itself, the content dimension was partitioned into *strands* at each of the three grade levels concerned. Each *strand* was itself partitioned into several *topics.* The cognitive behavior dimension was partitioned into three levels: computation, comprehension, and application/problem solving in much the same way as in previous mathematics assessments. The relative weighting of each strand at each grade level is summarized in Table 1.

Table 1. Weighting of strands in percent.

Strand	Grade 4	Grade 7	Math 10	Math 10A
Numbers & Operations	67	46	15	33
Data Analysis	8	12	15	17
Geometry	13	18	30	20
Measurement	12	12	10	15
Algebra	0	12	30	15

The table shows the percent of the total number of items assigned to each strand of content. For example, 12 percent of the items for Grade 7 were in the Data Analysis strand. Generally speaking, the weights attached to the content strands reflect the time allocated to those topics in the curriculum. Some strands or topics were assigned a larger proportion of items than their place in the curriculum might have suggested in order to provide a sufficient number of items in a given category to provide stable estimates of students' performance on that strand or topic.

Large numbers of achievement items in multiple-choice format corresponding to the cells of the table of specifications were assembled from a variety of sources. These included previous provincial assessments and similar evaluation programs in other jurisdictions. Many other items were developed expressly for use in this project, particularly for those strands and topics which had only recently been added to the mathematics curriculum. All of the newly developed items were pilot tested in the fall of 1989, and revisions or corrections were made as needed on the basis of those results.

The multiple-choice items were distributed among four test booklets for each grade level and each student responded to 40 items during the one hour allocated for the assessment. The booklets were balanced for difficulty as well as for content, and a timing pilot was carried out in a small number of classes to ensure that virtually every student would have sufficient time to respond to all of the items in his or her booklet.

Two booklets consisting of open-ended problem solving items were also prepared for each grade level, and these were administered to a provincially repre-

sentative sample of classes immediately following the regular assessment. These so-called *Q-forms* were also a feature of the 1985 assessment, and proved to be a valuable source of important information about students' achievement. The multiple-choice format is not a good vehicle for assessing areas such as problem solving, where students need to be given an opportunity to work toward a solution and not focus exclusively on selecting from the answers given.

All of the booklets included a number of items designed to obtain information about students' backgrounds, opinions, and attitudes. Students were also asked about several aspects of their classroom experience with mathematics, including their use of calculators and computers in mathematics classes.

Teacher Survey

Teachers of mathematics in Grades 4, 7, and 10 were asked to respond to comprehensive questionnaires developed for the study. The questionnaires were divided into five major sections dealing with aspects of the teachers' academic and professional backgrounds and experience, their opinions about a variety of topics included in the newly revised curriculum, their utilization of selected teaching practices, their implementation of the revised curriculum, and the availability of resources and professional development opportunities related to the new curriculum. They were also asked to indicate, for each of 20 items from the achievement tests administered to their students, their estimate of the percentage of their students who would get the item correct, and whether or not the mathematical content needed to respond correctly to the item had been taught. The latter is referred to throughout this report as *Opportunity to Learn*, a concept borrowed from the Second International Mathematics Study (Robitaille and Garden, 1989).

Goals of the Report

The 1990 Mathematics Assessment was designed to produce a portrait of the state of the teaching and learning of mathematics in all the schools of the province. The major goals of this report are to present our findings about students' achievement, their attitudes and opinions about a number of topics of contemporary interest in the field, and the opinions and attitudes of the teachers of mathematics. The report also includes a discussion of several areas of concern which the Contract Team wishes to bring to the attention of schools, school districts, and the Ministry of Education for their urgent consideration.

The report is divided into five sections, including this introduction. Part 2 deals with the students and teachers of mathematics and their attitudes and opinions about a number of issues concerning the teaching and learning of mathematics. Parts 3 and 4 present findings regarding students' performance on the achievement components of the assessment, and Part 5 includes a summary discussion of our conclusions.

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Mathematics is taught by the vast majority of elementary school teachers in the province, and by a large proportion of their secondary counterparts. All students are required to study mathematics every year from Grade 1 to Grade 11. No subject in the curriculum except English occupies as much of the teaching and learning energy of the educational system as does mathematics.

What do we know about those students and teachers, and about their attitudes and opinions concerning issues related to the teaching and learning of mathematics? Our findings in that regard are summarized in this section.

The Students

Virtually all students in Grades 4, 7, and 10 participated in the assessment. Most were enrolled in the regular program in English; approximately five percent were enrolled in either French immersion or in the *Programme-cadre de français.* For Grade 10 students, mathematics was usually offered as a full year, 10-month program; less than one-quarter were enrolled in a semestered mathematics course. Of those who were taking a Grade 10 mathematics course, about one-quarter were enrolled in Mathematics 10A, and about three-quarters were enrolled in Mathematics 10.

The future plans of Grade 10 students are very positive. More than half of them plan to continue their study of mathematics into their Grade 12 year. Almost 25 percent indicated that they planned to take Introductory Mathematics 11, suggesting that this course is seen as a viable route to Mathematics 11, which is required for most post-secondary programs. In addition, the majority of Grade 10 students plan to continue their education at the post-secondary level immediately after leaving secondary school or within the following year. Forty-five percent plan further study which is university-related, and another 27 percent plan study that is either business- or trade-related, or further study following a year out of school.

Students' Attitudes

The assessment sought not only to determine what mathematics students can do, but also what students think about various aspects of mathematics and instruction. Questionnaires were designed to determine students' attitudes and opinions about the value of mathematics in the workplace, about specific mathematics topics, and about how they learn mathematics in the classroom. Students' responses to these questions reflect how they perceive and respond to what happens in school, and the results provide further insight into the state of mathematics education in the province.

Mathematics and Jobs

Students responded to three questions concerning their beliefs about and attitudes toward mathematics and jobs. More than three-quarters of the students

at each grade level said that mathematics is needed to get a good job and that most people use mathematics in their jobs. If the majority of students believe that mathematics is needed for and used in desirable occupations, it is reasonable to expect that they might like a job that required mathematics. This was not the case. Slightly more than half the Grade 4 students, less than half the Grade 7 students, and only about a third of the Grade 10 students indicated that they would like to have a job that required the use of mathematics. These results reflect poor attitudes toward mathematics and declining interest in mathematics as students continue their study of it.

Topics in School Mathematics

Students' opinions about the content of their mathematics courses provide useful insights into the teaching and learning of the subject. At each grade level, specific topics were selected from the curriculum and students were asked how important and how easy they found the topic, and how much they liked it.

The majority of Grade 4 and Grade 7 students felt that all of the topics listed in the questionnaires were important. Students at all grade levels considered the traditional topics, such as basic facts and computation with whole numbers, fractions, or decimals, to be of greatest importance. The topics considered least important were: for Grade 4 students, using manipulative materials; for Grade 7 students, integers, geometry, data analysis, and estimation; and for Grade 10 students, trigonometry, exponents, and geometry .

Students generally identified strategies for problem solving as the most difficult topic. Grade 4 and Grade 7 students indicated that using calculators or computers was easiest, and Grade 10 students felt that basic number operations and area, perimeter, and volume were the easiest topics to learn.

Generally, students tended to like topics they found easy more than those they found difficult. However, Grade 4 students indicated that they liked using manipulative materials least of all of the mathematics topics listed, even though they rated it as relatively easy. This topic was also given the lowest importance rating. Grade 10 students tended to like familiar topics and those with obvious applications more than new or abstract topics.

Students' interest in and enjoyment of problem solving declined with grade level. About two-thirds of the Grade 4 students, half of the Grade 7 students, and one-third of the Grade 10 students found problem solving and learning problem-solving strategies enjoyable. Likewise, teachers at all three grade levels found problem solving difficult to teach and only moderately enjoyable. If implementation of the revised curriculum with its problem-solving focus is to be successful, much work needs to be done to provide teachers and students with the support they need to make problem solving less of an onerous task and more of an enjoyable adventure.

Classroom Practices

The revised curriculum includes a number of initiatives which could affect the way in which teachers and students engage in mathematical activ-

ities. The focus on problem solving and the recommended use of technological aids may influence the ways in which mathematics classrooms are organized. To gain some insight into the organization and activities of mathematics classrooms from the students' perspective, students were asked how frequently nine classroom activities occurred. These included the use of calculators and computers, as well as small group work.

According to results at all three grade levels, it appears that a typical mathematics class is highly traditional in nature with time allocated to review of homework, teacher lecture, and individual seat work with help from the teacher as needed. Students report that in a typical class manipulative materials are rarely or never used and that small group work seldom occurs. Research has shown that using concrete materials and encouraging cooperative or small group work may improve students' achievement and attitudes. The revised curriculum guide emphasizes the use of such practices, yet students' responses suggest that such practices do not occur on a regular basis in most classrooms.

Another emphasis in the revised curriculum concerns the use of calculators and computers. It is encouraging to note that Grade 10 students report frequent use of calculators in their mathematics classes, and this is a significant change since the last mathematics assessment. On the other hand, despite the strong curricular support for the use of calculators as instructional aids and as computational aids, about half of the Grade 4 and Grade 7 students reported that they rarely or never use calculators in their mathematics classes.

A Note on Course Selection in Grade 10

Students have a choice of two courses in Grade 10: Mathematics 10 and Mathematics 10A. The first is the Grade 10 mathematics course that students following the academic program usually take. It leads directly to Mathematics 11 and Mathematics 12, one or both of which are required for entry into a wide variety of programs at the post-secondary level. Mathematics 10A is a new course that was introduced to accommodate those students who have no desire to pursue any sort of post-secondary education that requires either Mathematics 11 or Mathematics 12 as prerequisites, or those who may be experiencing some difficulty with mathematics and wish to take an alternate route to Mathematics 11 and Mathematics 12. Thus, students who complete Mathematics 10A successfully may then take Introductory Mathematics 11, another new course, followed by Mathematics 11 and Mathematics 12, if they decide to continue.

Whenever academic choices must be made, the question of who makes the choice and on what basis that person makes the choice needs to be raised. Students (or their parents) who decide to take a course like Mathematics 10A should do so only in the full knowledge of the implications of that choice. For example, they need to know what options will be closed to them upon completion of secondary school if they elect to drop out of the academic program in mathematics.

The other side of this coin is that schools have to be fair in making recommendations to students about their options and in counselling them about course selection. There are no provincial guidelines for schools to follow in de-

ciding which students should go into Mathematics 10 and which into Mathematics 10A. This means that there may well be considerable variability among schools in the proportions of students who are advised to take courses such as Mathematics 10A. In some schools, the vast majority of students expect to continue their studies after high school. In that kind of school, there would likely be very few students enrolled in courses like Mathematics 10A. On the other hand, in another school where the proportion of students planning to continue their education is much lower, the numbers of students enrolled in Mathematics 10A might well be much higher.

We have no detailed information on school-by-school comparisons of enrollments in Mathematics 10 and Mathematics 10A, but we do know that about 25 percent of Grade 10 students are taking Mathematics 10A. We also know how those students' performance in mathematics compares to that of the students enrolled in Mathematics 10. That information is summarized in the boxplots shown below.

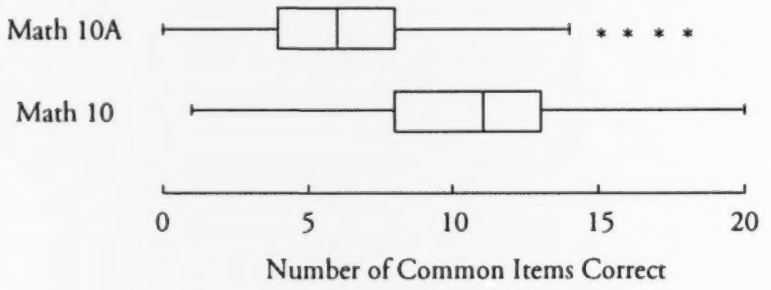


Figure 1. Box plot of achievement: Grade 10.

The boxplot for Mathematics 10 shows that Mathematics 10 students' scores on the 20 items common to both groups ranged from a low of 1 out of 20 correct to a high of 20 out of 20. The box in the middle of the distributions shows how the middle 50 percent of those students did, and the line in the middle of the box indicates the median. In other words, the 25th percentile for the Mathematics 10 group was 8 correct out of 20; the 75th percentile was 13; and the median was 11. The corresponding numbers for the Mathematics 10A students were 4, 8, and 6, respectively. The distribution for the Mathematics 10A group includes a number of so-called "outliers" whose performance, from a statistical point of view, was higher than expected for this group.

There is no doubt that, taken as a group, the Mathematics 10 students outperformed the Mathematics 10A students, nor is this unexpected given the nature of the two courses. What is surprising, and disturbing as well, is the extent to which the two distributions overlap. That is to say, there are many students enrolled in Mathematics 10A who had higher scores on this set of 20 items than many of the Mathematics 10 students. Why are these students not taking Mathematics 10? Unfortunately the assessment data cannot provide a response to this question, and we can only raise this as an issue which should be looked into. It may be, for example, that there is too great a degree of variability from school to school on the criteria used to decide who goes into Mathematics 10A.

Or, it may be the case that many students who could be successful at Mathematics 10 and subsequent courses have decided for reasons of their own to take the alternative route. Whatever the explanation, these results indicate that there may be a great waste of academic promise and talent involved in this situation, and a more complete investigation is needed.

Teachers

About half of the teachers who responded to the survey had more than ten years of teaching experience. At the same time, there has been a considerable influx of new teachers to the system since 1985, when the last mathematics assessment was conducted. At all three grade levels, the proportion of teachers with one to five years of teaching experience has increased substantially.

The vast majority of teachers of mathematics in the secondary schools of the province, over 80 percent, are male; the proportion of women teaching mathematics has increased by less than 5 percentage points in the past 10 years. The balance is slightly better at Grade 7, where about 35 percent of mathematics teachers are female. The Grade 4 percentages are the opposite of the Grade 7 figures: about 35 percent of Grade 4 teachers are male.

The sources of this disparity should be investigated to find out why so few women decide to pursue careers as teachers of mathematics. Enrollment patterns at the post-secondary level should be examined, because data from the Ministry of Education indicate that, at least in recent years, the proportions of male and female students enrolling in academic mathematics courses at the Grade 12 level are almost equal.

Some aspects of the academic preparation of teachers and of their involvement in professional activities are troubling. Approximately 60 percent of teachers indicated that they did not read professional journals frequently, and almost 40 percent said that they did not belong to a professional association other than the British Columbia Teachers' Federation.

Since 1985, there has been a slight decrease in the percentage of teachers who have taken three or more courses in mathematics or mathematics education as part of their professional preparation. In other words, there has been a decrease in the proportion of teachers who are most qualified to teach mathematics. Furthermore, over 20 percent of the Grade 4 and Grade 7 teachers indicated that they had never taken any post-secondary mathematics or mathematics education courses. Of the teachers of Mathematics 10A, about 20 percent have never taken a mathematics course at the university level, and over 40 percent have never taken a course in the teaching of mathematics. Concerns have been expressed in every mathematics assessment about the practice of assigning mathematics classes to teachers who have no specialized credentials, and particularly, as so often seems to be the case, assigning them those classes where students are most likely to need specialized assistance. Schools and school districts should take steps to remedy this situation and ensure that mathematics classes are taught only by teachers well-qualified to do so. In particular, school districts should encourage teachers to take, and universities should offer, courses in mathematics and mathematics education to meet this need.

Implementation of the Curriculum

The box plot in Figure 2 shows the variation in the amount of time teachers devote to the teaching of mathematics classes each week. The vertical line inside each box indicates the median, and the box itself represents the range from the 25th to the 75th percentile. The horizontal lines indicate the range of the data, except for the outliers, which are indicated by asterisks or small circles.

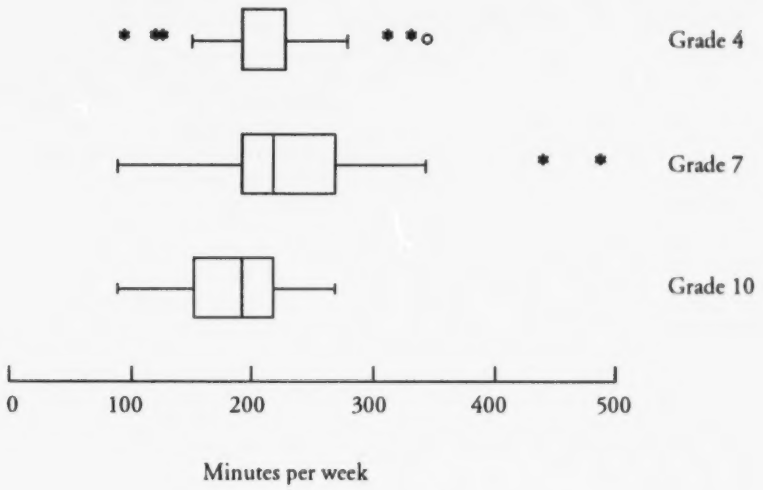


Figure 2. Number of minutes of mathematics instruction per week.

Overall, these results indicate that teachers are spending less class time on mathematics than they did in 1985. More than half of all teachers reported that their classes spent an average of less than 45 minutes a day on mathematics, and some reported spending as little as 20 minutes per day. It is interesting to note that many of these same teachers reported that the revised curriculum contains too much material. The apparent trend toward less time being devoted to the teaching of mathematics should be reversed.

There are some encouraging results concerning the implementation of the revised curriculum. Most teachers report that their teaching practices have changed in the past three years, coinciding with the introduction of the new program. Among the newly adopted or increasingly used practices, they cite increased attention to problem solving, cooperative learning, the use of calculators and manipulative materials, and new approaches to the evaluation of students' learning. The vast majority of teachers report that they have participated in one or more in-service programs in support of the revised curriculum in the past year, but about 25 percent indicated they were not aware that any such programs were available to them.

There is also good news in the area of materials to support the curriculum. Over 90 percent of teachers reported that they found the curriculum guide helpful to them and that they made use of it. A large majority agreed that the textbooks they were using matched the curriculum well and that the books were written at an appropriate level of difficulty for their students.

Opportunity to Learn

A number of the items used to measure students' achievement were also included in the teacher questionnaires in a section where teachers were asked to indicate whether or not their students had had the opportunity to learn (OTL) the mathematics content needed to answer the item correctly. All of the items dealt with mathematics content that is included in the prescribed curriculum for the grade concerned, but many of them dealt with topics that were either new to the curriculum or had been given increased emphasis.

The OTL results for Grade 4 are presented in Figure 3, and they are based on 57 items out of the total pool of 126 Grade 4 items. Averaging over all Grade 4 teachers and across these 57 items, 15 percent of the content was rated as "will be done in a subsequent year," or "will not be done." In the geometry strand, 21 percent of the content was rated in these categories.

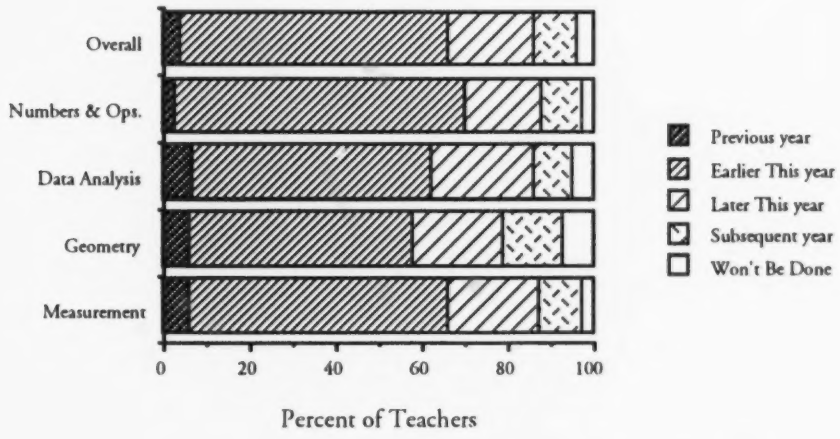


Figure 3. Opportunity to Learn, Grade 4.

It is not possible to tell from these data whether teachers think this content is inappropriate or whether they are not aware that it is included in the revised curriculum. However, it is disturbing that such substantial portions of the prescribed curriculum are not being implemented.

Furthermore, the assessment was administered in May at a time when only 15 percent of the school year remained; yet, 19 percent of the content was rated as "will be done later this year." This suggests that the teachers may have overestimated the extent to which they would be able to implement the curriculum.

The results from the OTL questionnaires for Grade 7 were similar to those for Grade 4. Averaging over all teachers and 58 selected items, 14 percent of the content was rated as "will be done in a subsequent year," or "will not be done." Twenty-one percent of the content was rated as "will be taught later in this school year."

The OTL results for Grade 10 suggest that teachers of Mathematics 10 and Mathematics 10A have more realistic estimates of the amount of content they will be able to cover in the remainder of the school year, about 10 per-

cent. The content that was rated as "will be taught in a subsequent year" or "will not be taught" was about 10 percent in the case of Mathematics 10A teachers and about 15 percent in the case of Mathematics 10 teachers. At the Grade 10 level there were marked differences across content areas. For example, in number and operations, measurement, and algebra only about 5 percent of the content was planned for teaching later in the current year; but in the geometry and data analysis strands, that figure was about 15 percent. Items on probability, a new topic for Grade 10, had the lowest OTL ratings of all.

Overall, the OTL results are rather disturbing. They indicate that, at all three grade levels, substantial proportions of the teachers had not taught a number of the new topics in the curriculum. In many cases, teachers indicated that the topic would be taught later in the year, but this seems unlikely given the time of year when the data were collected and the large numbers of topics involved. Even more worrying is the possibility that some teachers are not sufficiently familiar with the prescribed curriculum to recognize that the content needed to answer the items should be taught. There is a clear need for concerted action to ensure that all teachers are familiar with the prescribed curriculum and to provide them with the assistance and the materials they need to implement it as it was intended.

The major objective of the assessment was to evaluate students' achievement in mathematics at the level of the entire system in order to see how well the goals of the prescribed curriculum were being met. In Part 3 the achievement results are discussed in terms of what they tell us, in fairly general terms, about the kinds of mathematical tasks that students in our schools are able to perform. In the following section, they are discussed on a topic-by-topic basis at each grade level.

A Note on Methodology

Summarizing the vast amounts of information produced by a province-wide assessment for a report such as this necessarily results in the loss of a great deal of detail, and there is a danger that essential parts of the true picture will be lost. Moreover, the level of detail required for adequate reporting and understanding varies with the needs of the audience for the information. Thus, curriculum developers, researchers, teachers, educational policy makers, and elected officials all have very different requirements when it comes to understanding what the achievement results from an assessment mean.

In previous assessments, reporting of achievement results focused on the average percent correct for individual items, topics, and strands. Interpretation panels gave additional meaning to the numerical results by applying the labels "strong," "very satisfactory," "satisfactory," "marginal," and "weak" to describe students' performance. The writers of the assessment reports provided another set of interpretations of the achievement results on the basis of their experience and expertise in the area.

The interpretation panels for the 1990 assessment were not asked to evaluate students' performance using the "strong" to "weak" scale. Instead, a different approach was adopted to describe students' performance in global terms: an approach designed to focus attention on what students are able to do in mathematics. The goal was to provide a set of descriptors for four levels or categories of achievement at each grade level, and to divide students into those four groups on the basis of a statistical analysis of the results. In general terms, the steps of the procedure were as follows.

- A statistical estimate of the difficulty of each item was calculated.
- The items for each grade were ordered according to their calibrated difficulty values.
- The items were partitioned into four groups on the basis of their rankings in the previous step.

After the items had been partitioned in this way, the interpretation panels examined each set of items and made judgments about the mathematical skills and knowledge students needed to exhibit in order to attain the indicated results.

It is important to bear in mind what the following descriptions are intended to convey. To say, as we do in the next paragraph, that 95 percent of the Grade 4 students have been included in Category 1 does not mean that 95 percent of the Grade 4 students can do all of the things listed in the description of that category. What we can say is that 95 percent of Grade 4 students would be able to provide the correct solution for one or more of the Category 1 items 7

times out of ten. All of the skills or performances within a given category have been determined to be of approximately the same difficulty, and that is why they are grouped together. But no individual student was asked to respond to all of the items on which the category descriptions are based. Each student was given a test booklet containing about one-third of the total pool of test items.

What Mathematics Can Grade 4 Students Do?

The graph in Figure 4 shows the distribution of Grade 4 students across the four achievement categories defined by their performance on the achievement items. The lowest level, Category 1, includes 95 percent of the students. The next level in the hierarchy, Category 2, includes 59 percent. Categories 3 and 4 represent 21 and three percent, respectively.

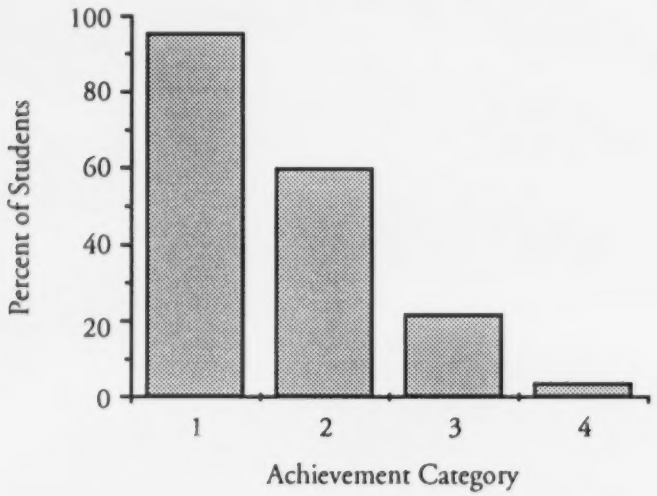


Figure 4. Distribution of Grade 4 students across achievement categories.

To say that 95 percent of Grade 4 students satisfy the Category 1 requirements means that 95 percent of Grade 4 students are able to supply the correct answer to one or more of the items belonging to Category 1, and that the probability that a Category 1 student would give the correct answer to any Category 1 item is 70 percent or more. Since the categories are hierarchical, all of the students in a given category also satisfy the requirements for all lower categories. Thus, the 3 percent of students in Category 4, for example, are also included in Categories 1, 2, and 3.

Each category includes a fairly wide variety of mathematical topics and skills. Listings of the skills and behaviors included in each category, along with the percentage of students assigned to the category, are given in the following paragraphs.

Achievement Category 1: 95 percent of Grade 4 students

Students in this category demonstrate an understanding of place value to 9999. They are able to perform basic operations (addition, subtraction, and multiplication), and to use patterning. They can read and interpret data from a simple graph and determine the appropriate use of linear units (centimetres and me-

tres). They demonstrate knowledge of geometry by identifying and analyzing attributes of some common plane and solid figures.

Achievement Category 2: 59 percent of Grade 4 students

In addition to the Category 1 abilities, these students demonstrate understanding of place value to 99 999. They are able to solve multi-step problems; to read, write, and order decimals in hundredths; and to recognize common fractions. They can read and interpret bar graphs and pictographs. They can identify labelled line segments, cylinders, congruent figures, and lines of symmetry, and can tile a region using a shape. They can identify the number of centimetres in a metre, determine the volume of a figure by counting the cubes shown in a pictorial representation, and solve problems involving capacity.

Achievement Category 3: 21 percent of Grade 4 students

In addition to the abilities in Categories 1 and 2, these students demonstrate understanding of the number system. They can solve problems involving division of whole numbers, addition and subtraction of decimals, application of fractions, and analysis of data. They can solve measurement problems involving area, capacity, mass, and time. They can relate plane figures to composite plane figures and to solid figures, and they can recognize a slide (translation).

Achievement Category 4: 3 percent of Grade 4 students

In addition to the abilities in Categories 1, 2, and 3, these students are able to estimate by comparing known quantities to new situations involving millimetres or kilometres. They demonstrate understanding of transformational geometry including rotations, and can find the areas of rectangles and triangles drawn on squared paper. They can use higher-level thinking to solve complex problems.

What Mathematics Can Grade 7 Students Do?

Figure 5 shows the distribution of Grade 7 students across the four achievement categories defined by their performance. The lowest achievement level, Category 1, includes 98 percent of the Grade 7 students. Category 2 includes 66 percent; Category 3, 40 percent; and, Category 4, 5 percent of the students.

Achievement Category 1: 98 percent of Grade 7 students

These students are able to solve exercises involving place value, subtraction of mixed numbers, and multiplication of whole numbers. They can rename numbers, fractions, or ratios. They understand properties of angles involving measurement and congruency, and can solve one-step problems involving length, volume, and capacity.

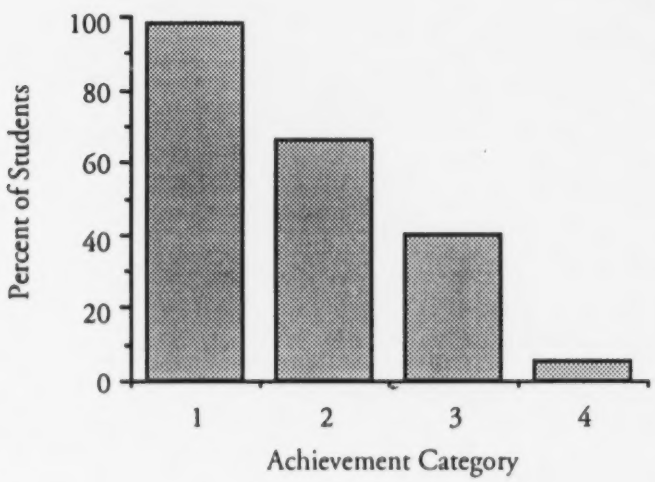


Figure 5. Distribution of Grade 7 students across achievement categories.

Achievement Category 2: 66 percent of Grade 7 students

In addition to the abilities in Category 1, these students are able to perform simple operations with proper fractions and sight multiplication with powers of ten. They can rename percents as fractions or decimals and use percent in problem solving. They can display and interpret data using bar graphs, broken line graphs, or circle graphs. They are able to apply geometric terms, use estimation skills to determine area and mass, and rename metric units. They can translate written statements into algebraic expressions and can solve whole number algebraic equations.

Achievement Category 3: 40 percent of Grade 7 students

In addition to the abilities in Categories 1 and 2, these students can use expanded notation, standard form, and estimation, and apply rules for order of operations with whole numbers and decimals. They can solve two-step problems involving ratio, multiples, area, or mass. They can calculate the area of rectangles and triangles and apply properties of symmetry and similarity. They can perform transformations and determine the coordinates of an image after a transformation. They can display data using frequency tables and interpret the median of a set of data. They can solve problems using charts, lists, and diagrams, and identify patterns. They are able to use inverse operations to solve percent and algebraic equations.

Achievement Category 4: 5 percent of Grade 7 students

In addition to the abilities in Categories 1, 2, and 3, these students are able to solve a variety of multi-step problems. They can identify the mode or median of a set of data and interpret data displayed in a stem-and-leaf plot. They can locate coordinates in a plane.

What Mathematics Can Grade 10 Students Do?

Grade 10 students can choose between two mathematics courses: Mathematics 10 and Mathematics 10A. Mathematics 10 is part of a sequence of courses leading directly to Mathematics 11, which is required for entry to most post-secondary institutions. Mathematics 10A is part of a sequence of courses intended for students who have experienced difficulty in mathematics. It provides an alternative for students requiring a slower pace of instruction and a delayed introduction of the more abstract content.

The content of the two courses are very different from one another, so it would make little sense to compare the performances of the two groups of students. For that reason, the interpretation panel defined separate sets of achievement categories for each course, and they are discussed separately here.

What mathematics can Mathematics 10 students do?

Figure 6 shows the distribution of Mathematics 10 students across the four achievement categories defined by their performance and described by the interpretation panel. Almost all students, 96 percent, satisfied the requirements for Category 1. Sixty-six percent satisfied the requirements for Category 2, 27 percent satisfied the requirements for Category 3, and 9 percent for Category 4.

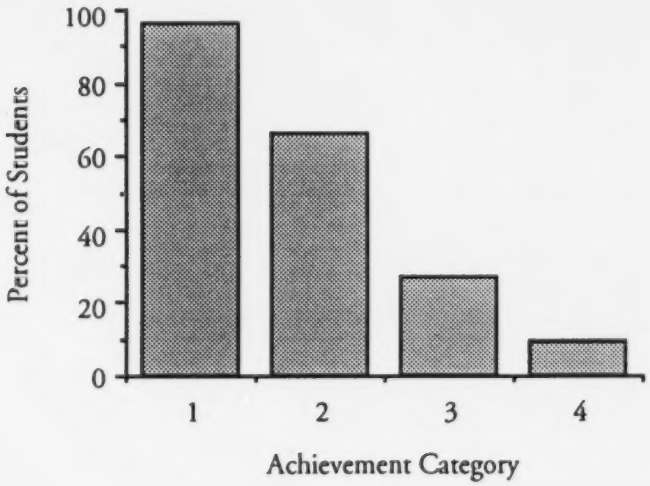


Figure 6. Distribution of Mathematics 10 students across achievement categories.

Achievement Category 1: 96 percent of Mathematics 10 students

Students in this category are able to read and interpret line graphs, solve basic measurement problems given a formula and a diagram, and perform basic algebraic operations. They can solve simple probability problems and can do multiple-step word problems involving whole numbers, money, and percent. They demonstrate a basic understanding of geometry involving properties of straight lines, triangles, and parallelograms.

Achievement Category 2: 66 percent of Mathematics 10 students

In addition to the abilities in Category 1, these students are able to perform operations, evaluate expressions, solve equations and inequalities, simplify and solve radicals, and estimate solutions to multi-step problems with rational numbers. They solve problems by interpreting tables, line, and bar graphs, and can find the probability of a single event. They solve measurement problems involving angles as well as plane and solid figures.

Achievement Category 3: 27 percent of Mathematics 10 students

In addition to the abilities in Categories 1 and 2, these students are able to perform operations on radicals and to solve problems in compound probability. They have an understanding of linear equations, and can solve problems involving similar triangles, right triangles, and transversals. They can find the areas of rectangles, triangles, and parallelograms, and the surface area of a rectangular prism. They can solve and graph inequalities and are able to solve multi-step problems in one variable and recognize when there is no solution.

Achievement Category 4: 9 percent of Mathematics 10 students

In addition to the abilities in Categories 1, 2, and 3, students in this category are able to interpret scientific notation displayed on a calculator and can recognize a number pattern when given a series of diagrams. Students can extrapolate information from graphs, tables, or lists of data and can solve probability problems involving restrictions. They are able to use algebraic skills to solve geometry problems and are able to solve multi-step quadratic equations. They are able to use the coordinate system in the Cartesian plane, the Pythagorean theorem, and use the basic trigonometric functions to solve right triangular problems. They can find the perimeter of composite figures involving circles and triangles and can use estimation skills for determining area.

What mathematics can Mathematics 10A students do?

The distribution of Mathematics 10A students across the four achievement categories, shown in Figure 7, is radically different from that of the Grade 4, Grade 7, or Mathematics 10 students previously discussed. Instead of almost all students being included in Category 1, only 78 percent of Mathematics 10A students are included in that level. This means, of course, that over 20 percent of these students failed to meet this minimal level of performance and underscores the fact that many students are not learning a sufficient amount of mathematics to cope with the demands of today's world. This is a situation which clearly calls for remedial action. Nineteen percent of the students are included in Category 2. Only 4 percent were able to meet the requirements for Category 3, and almost no students were included in Category 4.

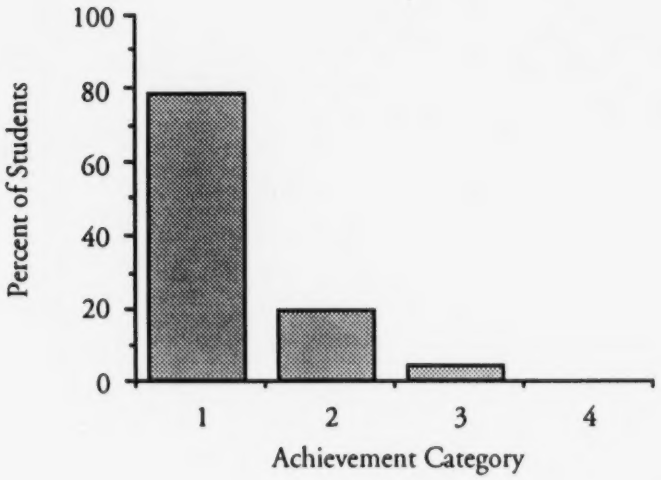


Figure 7. Distribution of Mathematics 10A students across achievement categories.

Achievement Category 1: 78 percent of Mathematics 10A students

These students are able to perform operations and solve consumer-related problems with positive rational numbers. They understand averages and interpret frequency tables as well as line and bar graphs. They are able to combine similar terms in algebraic expressions and can determine coordinates on a Cartesian plane.

Achievement Category 2: 19 percent of Mathematics 10A students

These students are also able to perform operations, evaluate expressions, and estimate solutions to multi-step problems with rational numbers. They can solve problems by interpreting tables as well as line and bar graphs, and can find the probability of a single event. They are able to solve measurement problems involving angles, plane and solid figures, and the Pythagorean Theorem.

Achievement Category 3: 4 percent of Mathematics 10A students

These students are able to perform operations with integral exponents and scientific notation, and to solve problems involving number patterns, proportions, means, and the probability of multiple events. They can solve triangles using the concept of similarity and trigonometric ratios. They can solve linear equations in one variable and convert word expressions into algebraic form. They are able to solve problems involving area, perimeter, composite figures, and estimation.

Achievement Category 4: 1 percent of Mathematics 10A students

These students are able to solve problems involving estimation and scientific notation, graphs and probability, complementary and supplementary angles, similar and right triangles, and trigonometric ratios. Students are able to simplify algebraic expressions and solve problems involving area and volume.

This section consists of a more detailed discussion of students' achievement. Their performance in each of the five strands (number and operations, data analysis, geometry, measurement, and algebra) and at each grade level is discussed with reference to selected examples of test items. In addition, students' responses to items assessing number sense and problem solving abilities are presented, as are changes in achievement since the last assessment in 1985.

Where test items are shown in the text, they are accompanied by an identifier of the format, Item 7B12. This refers to Item 12 from Form B of the Grade 7 booklets. All of the assessment items may be found in the appendices to the *Technical Report* which is available from the Ministry of Education.

Numbers and Operations

The development of number systems and of algorithms for efficiently carrying out operations on numbers are among our most significant achievements. Numbers and operations with numbers permeate every aspect of modern life, and they play essential roles in our political, social, scientific, and recreational activities. It is no exaggeration to say that students' understanding of number concepts and their ability to perform operations with numbers are essential to their understanding of mathematics and to their development as educated citizens.

The importance of numbers and operations is reflected in the emphasis given to these topics in the curriculum guide. As students progress from Grade 4 to Grade 10, they are given many opportunities to develop their understanding of numbers, their relationships, and their uses. For students in Grade 4, a good deal of the allotted time, about 60 percent, is dedicated to the development of such concepts as counting, place value, and theory of numbers, as well as to operations on whole numbers, decimals, and fractions. As students reach Grade 7 they continue to expand their knowledge of number concepts and operations through the introduction and development of topics such as ratio, proportion, percent, and integers. These topics enable students to explore the relationships among different representations of numbers, as well as to develop their proportional reasoning abilities. In the secondary school curriculum, students extend their knowledge of number and number operations to the irrational numbers and study their applications in algebra. The paragraphs which follow describe students' achievement in these topics.

Number Concepts

Number concepts such as place value, counting, estimation, and theory of numbers were assessed using items which focused on expanded notation, rounding, estimating, prime numbers, factors, and multiples. At the Grade 4 level emphasis was on place value, with some items assessing counting and basic concepts from number theory. At the Grade 7 level, items were selected to assess students' understanding of number theory through concepts such as order of operations, prime numbers, factors, and exponents. For Mathematics 10 and Mathematics 10A students, items which focused on order of operations and ex-

pressions involving exponents were used.

Place Value

Generally speaking, Grade 4 and Grade 7 students appear to have an adequate understanding of place value concepts. About two-thirds of the Grade 4 students demonstrated understanding of place value involving ones, tens, hundreds, and thousands. In particular, an impressively high percentage of students were able to select the standard form for 2000 + 700 + 40 + 8 as well as for 5 thousands + 8 tens + 6 ones. Some Grade 4 students found items involving ten thousands difficult. In determining what number is one thousand more than 39 934, almost half the Grade 4 students selected 40 934; but one third of them selected 139 934 as their response. Yet, virtually all Grade 4 teachers indicated that this content had been taught during the school year.

Grade 7 students demonstrated a good understanding of place value concepts involving decimals. For example, 80 percent of the students were able to identify the place value of 6 in 12.34567. Such results generally met the expectations of the interpretation panels.

Theory of Numbers

Number theory is an important component of the mathematics curriculum. Not only is it a source of interesting problems and beautiful patterns, but it has important applications as well. Children are fascinated by the regularities and patterns inherent in numbers, and exposure to topics in number theory provides them with opportunities to explore those phenomena and learn more about mathematics.

Factors, multiples, and prime numbers are among the important number theory concepts with which students had difficulty. For example, only slightly more than half the Grade 7 students were able to answer the items in this area correctly, even though most teachers indicated that they had taught this content. The Grade 7 Interpretation Panel was of the opinion that number theory is one of the least relevant and least significant topics in the curriculum. Therefore, although students' performance in this area was poor, it met the panel's expectations.

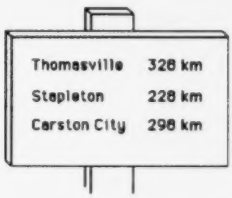
Simplifying numerical expressions using the rules for order of operations is an important topic in the curriculum, and the results in this area were also disappointing. Only about 10 percent of Grade 7 students correctly simplified the expression 3 + 4(5 + 2): about half of them selected 49 as the correct answer. Slightly more than half the Grade 10 students correctly simplified 30 - 4(8 - 2), whereas about one-third of them chose 156 as their response. While students' performance improved from Grade 7 to Grade 10, it was still below expectations. The interpretation panels recommended that teachers provide students with more opportunities to develop skill in applying these rules.

Estimation

Estimation is an important mathematical concept that permeates daily life and is emphasized throughout the curriculum. Grade 4 students were generally able to estimate quantities using comparison, but were less successful using estimation in problem situations such as the one illustrated in Item 4D05.

Item 4D05

Which two cities are about the same distance away?



		% of students	
A)	Thomasville and Stapleton	12	
B)	Stapleton and Carston City	30	
C)	Thomasville and Carston City	46	* (correct answer)
D)	All three cities are about the same distance away.	5	
E)	I don't know.	6	

Many Grade 4 students indicated that estimating was not only important, but also enjoyable and easy to learn. Capitalizing on this interest and providing increased opportunities for estimating quantities could improve students' estimating powers and sense for quantities.

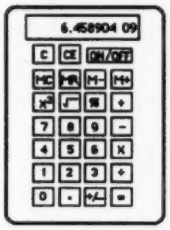
Grade 7 students had difficulty estimating the product of a series of decimal numbers. Only slightly more than half correctly estimated the product 9.75 x 11.134 x 2.9065 x 8.910, and a large proportion selected "I don't know" as their response. Yet, more than 90 percent of Grade 7 teachers stated that the mathematics required to answer this item had been taught. Grade 7 students, unlike those in Grade 4, considered estimation not to be important and not well liked. It appears that many students do not have a feeling for the power of computational estimation, a skill which is important in daily life.

Calculators

Using calculators and computers provides students with opportunities to perform complex computations and to increase their understanding of the number system. The use of calculators is encouraged at all grade levels in the revised curriculum, but this does not eliminate the need for students to become skilled in the use of standard algorithms for computation. It is encouraging to note that most Grade 10 students stated that they used calculators fairly regularly in mathematics classes. In spite of this, only slightly more than one-third of the students were able to correctly interpret the calculator display in Item 10A02.

Item 10A02

Someone has just multiplied 6540 by 987 600 on the calculator. The calculator with the answer displayed is shown below. The product of the two numbers is



		% of students	
A)	6 458 904 000	35	*
B)	6 458 904 009	10	
C)	$6.458\,904^{9}$	32	
D)	645 890 400	6	
E)	I don't know.	16	

More than half of Grade 4 students reported that they "rarely" or "never" used calculators during a typical school week. Interestingly, almost three-quarters of their teachers claim otherwise. Nonetheless, more than half of Grade 4 students were able to determine which buttons to push to solve problems using a calculator, but about 15 percent of students selected "I don't know" as their response. Overall the results suggest that students at all grade levels would benefit from increased experience in using calculators and also from activities designed to teach efficient calculator use.

Whole Number Operations

Whole number operations received considerable emphasis in the assessment, particularly at the Grade 4 level and to some extent in Grade 7. In general, Grade 4 and Grade 7 students demonstrated appropriate levels of skill in the application of routine operations involving addition, subtraction, multiplication, and division. Students were generally more successful on items involving multiplication than on those involving division. When number operations were embedded in situations or contexts such as word problems, Grade 4 students, in particular, had difficulty. For example, on Item 4B16 which is shown below, many Grade 4 students responded to the problem by simply adding or subtracting the

Item 4B16

Each page in a scrap book can hold 8 baseball cards. How many pages are needed to hold 60 baseball cards?

		% of students	
A)	7	17	
B)	52	20	
C)	8	30	*
D)	68	21	
E)	I don't know.	12	

numbers given without first thinking through the meaning of the problem situation. Such results suggest that problem-solving processes must be given increased prominence in mathematics teaching.

Overall, Grade 4 and Grade 10 students' performance in the area of whole number operations was lower than that expected by the interpretation panels. The Grade 7 panel felt that students at that grade level were generally performing at or above the expected levels.

Decimals and Fractions

As in previous assessments, results in the areas of decimals and fractions indicate that students generally had difficulty with these topics. At the Grade 4 level, operations with decimals is a new topic in the revised curriculum. This may explain why many Grade 4 teachers indicated that they had not yet taught the mathematics needed for students to answer the decimal items. Perhaps teachers feel that such concepts are too abstract for Grade 4 students.

Item 4A25

What fraction of the shapes are shaded?

		% of students	
A)	$3/6$	10	
B)	$2/3$	6	
C)	$3/9$	68	*
D)	$6/9$	4	
E)	I don't know.	10	

Note:
In the assessment booklets, fractions were written in vertical format.

Grade 4 students appeared to have difficulty with items involving hundredths, except when the context involved money. However, almost three-quarters of them were able to determine the decimal number represented by a point on the number line. Many Grade 4 students performed relatively better on fraction items which involved pictorial representations, for example Item 4A25, than on ones which relied on verbal presentations, for example Item 4D27.

Grade 7 and Grade 10 students performed better on decimal and fraction items embodying real-life situations, particularly when money was involved, than on items involving isolated computations. For example almost all Grade 7 students were able to correctly answer Item 7B10 which is shown on page 34.

On the whole, students' achievement on routine decimal applications met the expectations of the interpretation panels. However, Grade 7 students' performance on items involving common fraction operations generally fell below expected levels. For example, slightly more than half of Grade 7 students were able to correctly find the sum 1/2 + 1/3, but about one-third of them simply add-

Item 4D27

Tom has a collection of toy cars. He has 5 blue ones, 7 green ones, and 3 red ones. What fraction of his collection is green?

		% of students	
A)	$7/10$	14	
B)	$7/15$	46	*
C)	$7/8$	23	
D)	$8/7$	5	
E)	I don't know.	9	

Note:
In the assessment booklets, fractions were written in vertical format.

ed the numerators and denominators together. This poor performance could be due to a decreased emphasis on fractions in the revised curriculum. Although such operations have been de-emphasized, they have not been removed from the Grade 7 curriculum. They are still important, and they do require attention.

Item 7B10

There are 25 students in Mary's class. The class goes on a field trip to the museum. If tickets cost $2.85 each, what is the total cost of tickets for the class?

		% of students	
A)	$8.77	1	
B)	$19.95	3	
C)	$71.25	91	*
D)	$91.20	3	
E)	I don't know	2	

Ratio, Proportion, Percent

Grade 7 students and Mathematics 10A students responded to items dealing with ratio, proportion, and percent. Items assessed students' ability to find equivalent ratios and to convert among fractions, decimals, and percent. They also applied such concepts in word problem situations. In general, students appeared to have an understanding of equivalent ratios but were somewhat less proficient at performing conversions among decimals, fractions, and percents. They were more successful calculating a percent of a number than at solving other types of percent problems. The interpretation panel was pleased with students' achievement in this area, but they noted that relationships between ratio, proportion, and percent need further emphasis.

Extending the Number System

The topics of integers and irrational numbers were assessed at the Grade 7 and Grade 10 levels. The topic of integers is new in the revised curriculum for Grade 7 students, and this may explain why more than half of Grade 7 teachers indicated that they had not yet taught this topic. In general, Grade 7 students performed better on items which involved a real-life context or application than they did on isolated integer computations. This indicates the importance of situated learning. As a result, the Grade 7 interpretation panel suggested that integers be taught with reference to real-life situations as much as possible.

In general, Mathematics 10 students found operations involving radical expressions difficult, and in most cases the results did not reach the levels expected by the interpretation panel. All items involved isolated computations, that is, the items were presented without a context or problem situation. Perhaps performance would have been higher if students had been given the opportunity to experience the nature of irrational numbers and their relationship to number systems in problem situations.

Data Analysis

Collecting, organizing, and analyzing data, and making predictions are normal activities in the course of everyday life. Many decisions are made on the basis of an analysis of numbers within particular situations or contexts. From personal decisions such as planning an overnight camping trip when the weather forecast predicts a 20 percent chance of rain, to societal decisions such as those made by political parties on the basis of opinion polls, data analysis plays an important role. Students need to be confident about their ability to use and interpret information in making sound decisions and judgements. An understanding of data analysis, its power and use, is therefore important for all students.

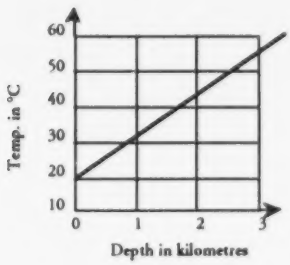
The inclusion of data analysis in the revised curriculum is a reflection of this importance. In Grade 4, students are given opportunities to build on their intuitive experiences of quantitative reasoning by exploring ways in which data may be collected, organized, displayed, and interpreted. In Grade 7, students further enhance their understanding of data and uncertainty through discussions about the use and misuse of statistical information, the reasonableness of results, statistical bias, and basic probability concepts. Building on these exploratory concepts, Grade 10 students deepen their understanding of data analysis concepts through continued experience in collecting data and using it for prediction and decision making in various practical situations. For the assessment, students responded to items that dealt with organizing data, reading and interpreting various kinds of graphs and tables, and predicting outcomes.

Information that requires analysis and interpretation for decision making is often presented in the form of pictures or graphs. Collecting, organizing, and interpreting information in graphs or tables as well as dealing with notions of uncertainty are therefore important skills for the development of statistical thinking. Students at each grade level responded to items that assessed such statistical reasoning, and the results indicate some general strengths as well as some areas of concern.

Students at all three grade levels were generally able to collect and interpret data presented in graphs and tables in order to determine single values or direct relationships. For example, 88 percent of Grade 10 students responded correctly to Item 10C06.

Item 10C06

From the graph below, the temperature at a depth of 2.5 km is closest to



		% of students		
		All Grade 10	Math 10A	Math 10
A)	30°C	3	6	2
B)	40°C	5	8	4
C)	50°C	88	78	91 *
D)	60°C	2	5	1
E)	I don't know.	2	3	1

Many Grade 7 students were able not only to interpret graphs and tables but also to use the information to solve applied problems. About two-thirds of Grade 4 students were able to select the appropriate graph for a given set of data.

It is encouraging to find that many teachers have begun to introduce some data analysis concepts in their teaching. However, data analysis seems frequently to be left to the end of the school year or not taught at all. For example, almost half of the Grade 7 teachers indicated that the topics of mean and median had not yet been taught. Almost all Grade 7 teachers had not yet taught the construction and interpretation of stem-and-leaf plots. About one-third of Mathematics 10A teachers indicated that the concept of mean had not been taught, although most teachers reported that the concept of average had been taught. Almost half of the Grade 4 teachers indicated that the content needed to respond correctly to an item requiring a conceptual understanding of data collection had not yet been taught. It is therefore not surprising that students did rather poorly on items involving these topics.

A number of items at the Grade 7 and Grade 10 levels assessed students' understanding of some basic concepts and methods dealing with probability. In

general, students performed poorly on these probability items, and they had difficulty both with items involving single events and those concerned with compound events. Again this is not surprising considering the fact that more than half the teachers stated that they had not yet taught the content in this area. It is interesting to note, however, that research has shown that students' intuitive notions of uncertainty often contradict the formal and accepted laws of probability which describe various events. Students would therefore benefit from additional experience with probability topics related to their own intuitive understandings of uncertainty.

The interpretation panels at all three grade levels recognized that data analysis is new to the revised curriculum and that it takes time for teachers to implement changes. In recognition of the importance of statistical reasoning in today's society, the interpretation panels encouraged teachers to continue to emphasize data analysis, suggesting that these topics be integrated throughout the curriculum. Data analysis can provide realistic settings for the development and application of ratios, fractions, decimals, and percent, as well as providing opportunities for students to make conjectures and design experiments.

Geometry

Geometry, the study of shape and space, provides many opportunities for students to make sense of the physical world. Knowledge and investigation of geometric ideas, properties, and relationships are important for use in many different mathematical and real-life situations.

The study of geometry from Grades 4 through 10 develops from informal intuitive explorations to formalized, structured investigations. At all grade levels geometry is more than simply naming and recognizing shapes. It includes analyzing shapes and the relationships among their parts, together with a focus on investigating geometric ideas and developing spatial sense. The study of geometry can act as a vehicle through which students make connections to other areas of mathematics such as number operations, measurement in two and three dimensions, problem solving strategies, and patterns.

Despite the recognized importance of geometric ideas and relationships, many students in British Columbia had not yet been given the opportunity to explore geometric topics by the time of the assessment. The data from the teacher questionnaires indicate that about half of the Grade 4 and Grade 7 teachers and one-third of the Grade 10 teachers responded that the mathematics needed to answer the geometry items correctly had not been taught. It is therefore not surprising that student achievement on many of the geometry topics was below expected levels. Nonetheless, students generally demonstrated competence in those areas which had been taught.

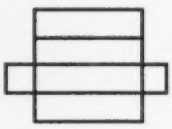
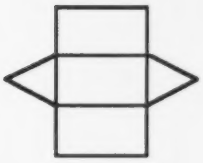
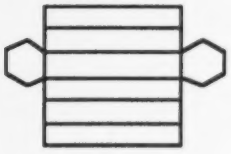
Properties

Knowledge about geometric shapes and properties enables students to communicate and reason about their experiences with objects. The results of the assessment indicate that students were able to identify various geometric shapes.

For example, most Grade 4 students were able to identify solids such as cubes, cones, and cylinders. They were also able to select the pattern or net used in making various solids, as illustrated in Item 4B32.

Item 4B32

Which of these figures can be cut out and folded to make a pyramid?

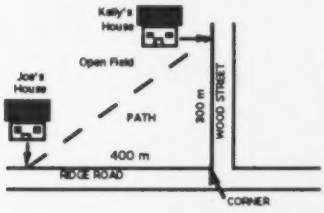
		% of students			% of students
A)		78 *	B)		3
C)		8	D)		3
E)	I don't know	4			

Grade 7 students were generally able to identify congruent angles, use a protractor to measure angles, select the definition of an equilateral triangle, and state the number of edges on a cube. Both Mathematics 10 and 10A students were able to identify vertical angles and supplementary angles.

Students had difficulty with some geometric properties. For example, less than 20 percent of Grade 4 students selected the diagram of a circle as the shape formed by a path that is a constant distance from a fixed point. Only 17 percent of Grade 7 students were able to determine the number of pairs of parallel planes in a right rectangular prism. As Item 10A11 illustrates, only about half of Grade 10 students were able to apply the Pythagorean Theorem even though almost all the Grade 10 teachers indicated that this content had been taught.

Item 10A11

When Joe walks from his house to Kelly's house, he follows the path through the open field. How far does he walk?



		% of students	
A)	450 m	16	
B)	500 m	56	*
C)	550 m	10	
D)	600 m	10	
E)	I don't know.	8	

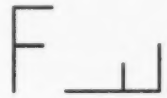
Transformations and Relations

Students' spatial reasoning abilities were assessed through items involving transformational geometry, tessellations, symmetry, and relations among various shapes and figures. Grade 4 students were generally able to identify congruent pairs of figures and were familiar with the basic concept of tessellation. Both Grade 4 and Grade 7 students had some difficulty with items involving transformations, Item 7B30 for example.

Item 7B30

In which one of the following diagrams is the second figure a rotation of the first?

A)

B) 

C)

D)

E) I don't know.

		% of students	
A)	A	21	
B)	B	12	
C)	C	36	*
D)	D	24	
E)	I don't know	6	

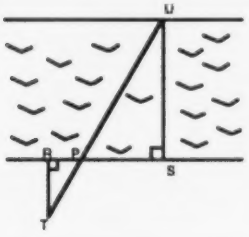
The disappointing performance on these items is not surprising considering that a large proportion of Grade 4 teachers, about 20 percent, and Grade 7 teachers, about 30 percent, indicated that the content needed had not been and would not be taught, although it is included in the curriculum. This is an area of concern, since research has shown that the development of spatial reasoning abilities is an important factor in the the general intellectual growth of students, as well as in their mathematical development.

Basic concepts and applications of similar figures were difficult for students in Grade 7, Mathematics 10, and Mathematics 10A. Although almost 60 percent of Mathematics 10 students were able to identify properties of similar triangles, they were generally less successful at using those properties to solve applied problems such as the one in Item 10C49, an item which closely matches a limiting example found in the curriculum guide.

The concept of similarity is extended for Grade 10 students with the introduction of trigonometry, a new topic in the revised curriculum. Both Mathematics 10 and Mathematics 10A students are expected to be able to identify and use trigonometric ratios to solve problems involving right triangles. At the time of the assessment more than half of the Mathematics 10A teachers indicated that

Item 10C49

The figure below illustrates a water canal and a method of measuring its width. If PS = 24 m, PR = 2 m, and RT = 5 m, how wide is the canal?



		% of students	
A)	24 m	5	
B)	32 m	17	
C)	40 m	17	
D)	60 m	38	*
E)	I don't know.	22	

this topic had not yet been taught, but about one-third of them reported that the content would be taught later in the current year. It is therefore not surprising that a large proportion of students responded "I don't know" to the trigonometry items, and that student achievement in this area was much lower than expected.

Analytic Geometry and Proof

Coordinate graphing is a topic that was assessed at all grade levels, and it is a new topic in the revised curriculum for Grades 4 and 7. A large proportion of students at each grade level made the common error of interchanging the order of the x- and y- coordinates on items requiring them to determine the coordinates of selected points on a grid. Mathematics 10 students were generally able to recognize the graph of a linear equation, but were less successful on items involving systems of linear equations.

Mathematics 10 students also responded to a number of items involving geometric proofs. Performance in this area was much higher than expected, and this suggests that the development of geometric reasoning through formal proofs is receiving appropriate attention.

Measurement

The development of measurement concepts from Grade 4 to Grade 10 progresses from informal explorations of basic measurement concepts toward more formal applications of skills in new situations. Selecting appropriate units of measure, knowing the relationship between units, and estimating measurements were assessed within the topic areas of length, perimeter, area, volume, capacity, and mass.

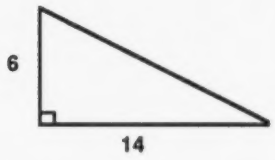
Grade 7 students performed particularly well on items involving estimation: the thickness of a dime, the area of a classroom, and the mass of a grain of sand. However, less than half the Grade 4 students were able to correctly estimate the length of a paper clip in millimetres, and only about two-thirds of them were able to select the appropriate unit for measuring the length of a garden

fence. Such results suggest that Grade 4 students lack a basic conceptual foundation for measurement, a foundation which is necessary for the development of further measurement techniques.

Grade 7 and 10 students also demonstrated a lack of conceptual understanding, focusing more on applying procedures or algorithms regardless of the context. Item 7C33 is an example of this apparent lack of conceptual understanding. For this item only slightly more than one-third of the Grade 7 students were able to find the area of a right triangle, while about the same proportion of students selected the response determined by multiplying the lengths of the legs of the triangle.

Item 7C33

Find the area of this right triangle.

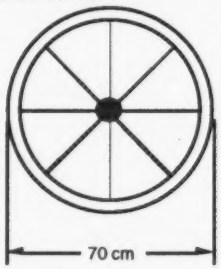


		% of students	
A)	42	36	*
B)	20	14	
C)	84	40	
D)	21	4	
E)	I don't know.	5	

Most Mathematics 10A students demonstrated a lack of understanding of the relationship between the circumference and diameter of a circle. As can be seen from Item 10B36, a surprisingly large percentage of them, almost half, selected the response determined by doubling the diameter.

Item 10B36

The diameter of a tricycle wheel is 70 cm. Approximately how far forward does the tricycle travel for one complete turn of its wheel?



		% of students	
A)	70 cm	15	
B)	140 cm	44	
C)	220 cm	22	*
D)	3850 cm	8	
E)	I don't know.	8	

In general, students did better on items which included the dimensions of the figure and asked for the perimeter, area, or volume, as compared to items where one or more dimensions were unknown. Moreover, students performed better on one-step problems than on multi-step problems. Item 4C39 illustrates the difficulty students had with multi-step problems.

Although the majority of teachers at all three grade levels stated that the content for this strand had been taught, students' achievement was generally be-

Item 4C39

Bella can jump 627 cm high. Dana can jump 5 m high. How much higher can Bella jump than Dana?

		% of students	
A)	127 cm	37	*
B)	622 cm	20	
C)	22 cm	16	
D)	632 cm	8	
E)	I don't know.	13	

low expected levels in many topic areas. The teaching of measurement concepts should be based on the active exploration of students' everyday experiences and should focus on the meaning of measurement in various contexts.

Algebra

As a generalization and extension of the number system, algebra is a key ingredient in the reasoning processes involved in thinking and communicating about mathematics. An understanding of algebraic concepts and their use enables students to construct, solve, and interpret mathematical models of problem situations.

The introduction and development of algebraic concepts is a new topic in the revised curriculum for Grade 7. At this level students' experience with algebra is informal, building on their understanding of number concepts. As students progress, a more formalized approach is developed so that they are able to communicate using the language of algebra and solve a variety of problems in different contexts.

Students in Grades 7 and 10 responded to items that assessed their ability to work with algebraic expressions and equations. In general, the achievement results at both levels were lower than expected. The majority of Grade 7 students were able to solve simple equations of the form $n + 3 = 9$ or $17x = 119$, yet were less successful at solving equations of the form $x/8 = 16$, even though most teachers indicated that such content had been taught. Students generally found the construction of algebraic expressions and equations from verbal expressions to be difficult, as Item 7A36 illustrates.

Item 7A36

Which one of the following stands for the product of a number and 6?

		% of students	
A)	$y + 6$	25	
B)	$y - 6$	5	
C)	$6y$	42	*
D)	$y/6$	13	
E)	I don't know.	13	

Note:
In the assessment booklets, fractions were written in vertical format.

Most Grade 10 students were able to simplify expressions and solve equations containing positive integral coefficients without parentheses. They appeared to have more difficulty with items involving expressions or equations with negative numbers, decimals, fractions, or parentheses. Grade 10 students found translating a word phrase into an algebraic statement and then solving it for the unknown to be difficult. This can be seen in Item 10D19.

Item 10D19

If five is added to a certain number and the sum is multiplied by three, the result is –17. Find the number.

		% of students		
		All Grade 10	Math 10A	Math 10
A)	$-2/3$	16	14	17
B)	4	11	17	8
C)	$-7\ 1/3$	15	18	14
D)	$-10\ 2/3$	33	16	40
E)	I don't know.	24	34	20

Note:
In the assessment booklets, fractions were written in vertical format.

The development of algebraic skills is an important foundation for the understanding and communication of further mathematical ideas. However, an emphasis on symbol manipulation in isolation from problem situations will likely do little to improve students' understanding and use of algebraic processes in varied contexts. Algebraic skills can be developed and enhanced through their application in other strands such as data analysis, geometry, and measurement. Through such contexts students may gain an appreciation for the power and use of algebra.

Number Sense

Number sense refers to a general feeling of confidence about numbers, their uses, relationships, and meanings. A sense of numbers is needed in order for students to work meaningfully with the quantitative information found in everyday situations. Students' ability to produce reasonable estimates, to choose appropriate calculating tools and procedures, to judge the reasonableness of results, to detect calculating errors, and to understand various relationships between numbers and operations are all important aspects of what it means to have a sense for numbers. Number sense is not a separate curricular strand, but instead may be considered as a way of thinking about numbers that should permeate all mathematical topics.

For this assessment students' number sense was assessed at all grade levels using the same items. These items were designed to assess students' ability to

produce reasonable estimates with numbers of different magnitudes in varied contexts and their understanding of relationships among numbers. Here are two examples.

Number sense items

About how much does a horse weigh?

		% of students			
		Grade 4	Grade 7	Grade 10	
A)	4 kg	2	2	2	
B)	40 kg	8	7	5	
C)	400 kg	46	62	61	*
D)	4000 kg	30	24	17	
E)	I don't know.	8	7	15	

How high would a stack of one million pennies be?

		% of students			
		Grade 4	Grade 7	Grade 10	
A)	2 m	32	25	11	
B)	200 m	22	21	17	
C)	2 000 m	18	19	25	*
D)	20 000 m	13	13	16	
E)	I don't know.	14	21	30	

The results suggest that students at all three grade levels have a better sense for numbers found in familiar contexts that are close to their personal experiences than for numbers outside those contexts. For example, less than 25 percent of students at each grade level were able to select a reasonable estimate for the height of a stack of one million pennies. They were much more successful at estimating the mass of a horse.

The cross-grade results for these items suggest that students' ability to produce reasonable estimates in familiar contexts improves with grade level, in particular between Grade 4 and Grade 7. That there is little difference in the results from Grade 7 to Grade 10 suggests that older students may be given fewer opportunities to develop a sense for the magnitude of numbers in varied contexts. Developing a sense for the magnitude of large numbers such as one million appears to be difficult for students at all grade levels, but it is important. Such numbers are used regularly in discussing political, geographical, economic, and technological issues; yet it is often difficult to imagine or visualize them. Number sense develops with experience, and students should be given more opportunities to develop this skill.

Problem Solving

Learning to solve problems is an important part of learning mathematics. Through attempts to solve problems students are able to investigate, explore, and experience the nature of mathematics. Hence, problem solving is an important part of the revised curriculum. For students in Grades 1 through 7, the curriculum identifies problem solving as a separate strand in order to encourage the development and use of specific problem solving skills and strategies. Beyond Grade 7 the emphasis on problem solving is continued; however, it appears, not as a separate strand, but as an integrated component of all strands. Developing students' abilities to select and implement problem-solving strategies in different problem situations is clearly a prominent feature of the revised curriculum at all grade levels.

For this assessment students responded to a variety of multiple-choice problem-solving items that were distributed across all of the content strands. These non-routine problems were designed in the expectation that students would understand the problem and recognize the possibility of a solution for each problem, but would not have a standard procedure or routine to solve it. Students would therefore be encouraged to use a problem-solving strategy such as looking for a pattern, making a table or organized list, or solving a simpler problem.

Problems presented in multiple-choice format provide information on the types of answers students select but give little information about the strategy or approach taken to solve the problems. A set of open-ended problem-solving items was therefore administered to obtain information on students' solution approach and on the degree to which they were able to use the problem-solving strategies outlined in the curriculum guide. These items were assembled into forms identified by the letter Q and administered to representative samples of students. Together with the multiple-choice items, they provide a good deal of information about both the students' rate of success and the solution processes they used in solving the problems.

The percent correct for the multiple-choice problem-solving items at both Grade 4 and Grade 7 was low; generally less than half the students answered any item correctly. Given the inherent difficulty of the items, and the fact that such problems are difficult to solve under time constraints, this result is not surprising. Problem solving is a complex process, and correct solutions cannot be expected with the same frequency as in the case of computational exercises or other routine mathematical tasks. It is encouraging to note, however, that Grade 4 and Grade 7 teachers' ratings of the opportunity to learn for such items was generally more than 70 percent. This information coupled with the low success rate of students supports statements by teachers that problem solving is one of the most difficult topics to teach and to learn.

Item 4B18 is an example of a non-routine problem in multiple-choice format given to Grade 4 students. This item required students to detect and extend a pattern, and using a table would certainly have been a useful strategy. Although 73 percent of teachers indicated that they had taught the content needed to answer this item correctly, only about a third of students selected the correct

answer. An additional third chose 13 pages as their response, perhaps because they added three for each of the two days. Because students were given this problem in a multiple-choice item, it is not possible to tell what operations students used in working this problem, or whether they made a list or table.

Item 4B18

On the first day Joe read one page of a book, on the second day he read 2 pages, on the third he read 4 pages, and on the fourth day he read 7 pages. If Joe continued to read the book following this pattern, how many pages did he read on the sixth day.

		% of students	
A)	32	9	
B)	31	9	
C)	13	34	
D)	16	34	*
E)	I don't know	13	

The open-ended problems shed more light on students' selection and implementation of strategies in working toward a solution. The results indicate that about 50 percent of students were at least able to select and initiate an appropriate strategy for solving a problem. Moreover, about one-third of students were able to select and implement an appropriate strategy with no errors or only minor errors in the solution process. Considering the level of complexity of these problems, this result is encouraging.

About one-quarter of students at each grade level showed no work or identified only the data in their attempts to solve problems, and this is a cause for concern. It was expected that all students would find some problems difficult and challenging; however, it appears that many students often find problem solving too formidable a task even to attempt. This may be due to the difficulty of the problems, students' inexperience in solving problems, an unwillingness to attempt such problems, or a lack of familiarity with problem-solving strategies.

Students were generally more successful at solving problems requiring direct calculations than those requiring general strategies such as guessing and testing or making a systematic list. These general strategies were used less frequently than expected. For example, on a complex problem given to Grade 7 students where the strategy of guess and test was most appropriate, only 4 percent of students used that strategy while 36 percent used numerical operations without guessing and testing. Such results suggest that students need increased opportunities to solve problems requiring general strategies, including challenging problems where the appropriate strategy is not obvious.

The strategy of choosing numerical operations in a seemingly arbitrary manner was used excessively by many students. For example, on a problem given to Grade 10 students in which using guess and test or using algebra was considered appropriate, 34 percent of students tried number operations, while only 17 percent used algebra, and 22 percent used guess and test. As students' experience with solving problems increases, so too might their insight into the degree of ap-

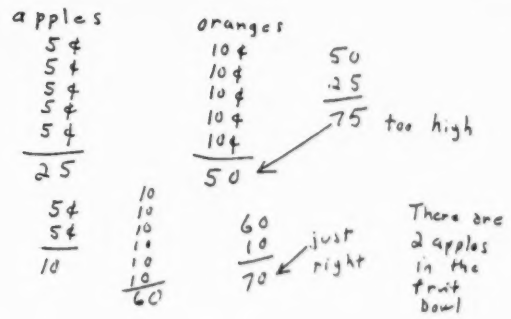
propriateness of specific strategies for particular problem situations.

Solving problems requires awareness of problem situations, decision making, and continuous self-regulation or critical analysis of strategy implementation. To assess critical analysis in problem solving, students were given an already solved problem containing a significant error, and asked for their thoughts on the way the problem was solved, the appropriateness of the strategy used, and the reasonableness of the answer. Students at each of the three grade levels responded to items presenting a solved problem, similar to Item 7Q1-7 which is shown below.

Item 7Q1-7

A bowl contains 10 apples and oranges.
Apples cost 5 cents each and oranges cost 10 cents each.
All together the fruit is worth 70 cents.
We want to find how many apples are in the bowl.

Beth tried to solve the problem this way.



Try to follow Beth's work and solution. Then answer these questions:

Is Beth's way of solving the problem a good one? Tell why you think it is or is not a good way.

Did Beth get the right answer? Explain why she did or did not.

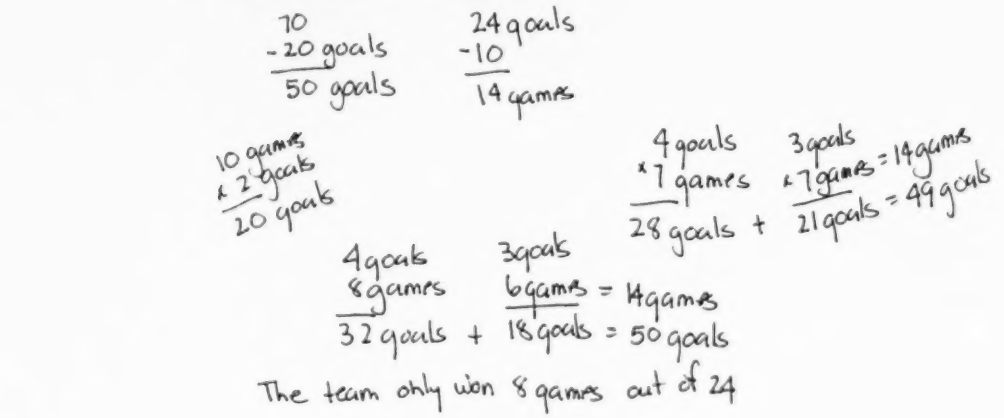
Most students' responses to these problems were simplistic, lacking evidence of attempts to comprehend or critique the solution, and indicated that students appear to have little or no experience in critically analyzing problem solutions. Critical evaluation of all aspects of the solution process is an important part of problem solving, and it appears that students require further experience in this area. In the classroom, problems of this type provide students and teachers with rich opportunities to discuss the selection and implementation of appropriate strategies. Such problems also have the potential to promote students' self-monitoring and evaluating of their problem-solving efforts.

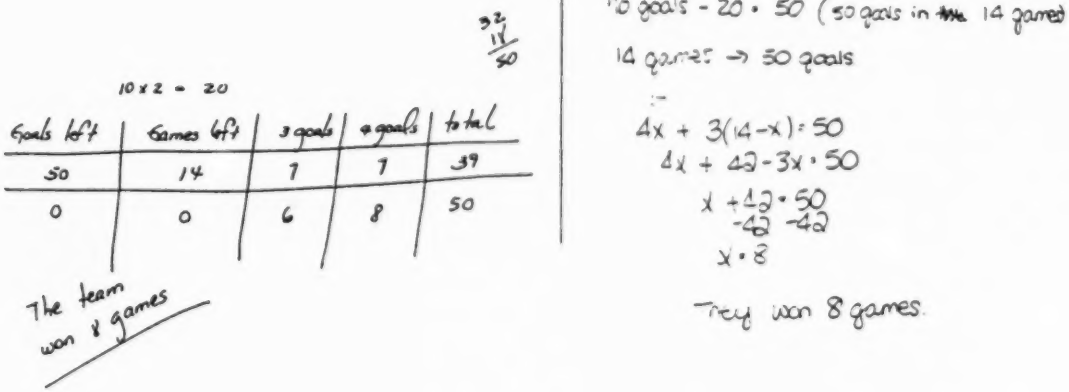
A number of problems were common to Grades 7 and 10, and there was one problem that was common across all three grade levels. The results indicate that experience and maturity contribute significantly to success in problem solving. On six problems given to both Grade 7 and Grade 10 students the achievement of Grade 10 students was considerably higher than that of Grade 7 students. However, at both grade levels various strategies were selected and

implemented as can be seen in the samples of different students' attempts to solve the hockey problem in Item 7Q1-6/10Q1-5.

Item 7Q1-6
Item 10Q1-5

A hockey team scored 70 goals in 24 games.
In 10 of the games the team scored 2 goals in each game.
In the other games they scored either 3 goals or 4 goals.
The team won only when they scored 4 goals.
How many games did the team win?




32
11
50
10 x 2 = 20
Goals left | Games left | 3 goals | 4 goals | total
50 | 14 | 7 | 7 | 39
0 | 0 | 6 | 8 | 50
The team won 8 games
70 goals - 20 = 50 (50 goals in the 14 games)
14 games → 50 goals
4x + 3(14-x) = 50
4x + 42 - 3x = 50
x + 42 = 50
-42 -42
x = 8
They won 8 games.

In general, the results from both the multiple-choice and open-ended items suggest that students find solving problems difficult. Students need to recognize that some degree of difficulty is inherent in the nature of problem solving. It is through perseverance, continued experience solving a variety of problems, and making use of opportunities to discuss and share ideas about solutions and methods that students will become confident problem solvers. This is not an easy task, as is underscored by the fact that most Grade 7 and Grade 10 teachers

reported that problem solving is the most difficult but also an enjoyable topic to teach.

Changes in Achievement

A number of items from the 1985 assessment were included in the 1990 item pool to investigate patterns of change in students' achievement over time. These results are summarized in Figure 8.

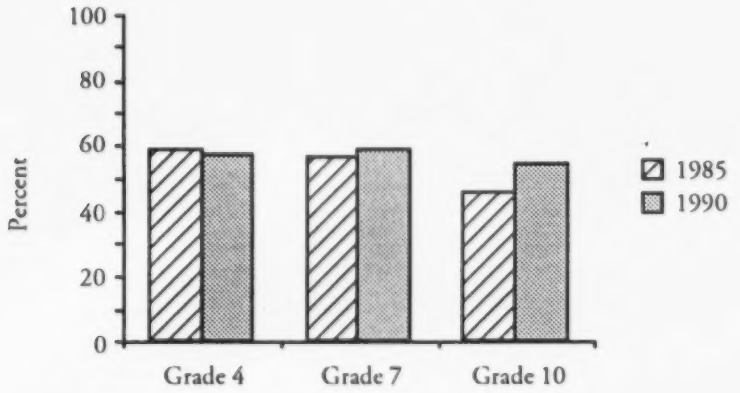


Figure 8. Change in student achievement 1985-1990.

Overall changes were very small at both Grades 4 and 7. The changes are on the order of one less item correct on a 50-item test in 1990 in the case of Grade 4, and one more item correct on a similar test in the case of Grade 7. A number of individual items showed more dramatic changes, but, taken together, these results show that there has been little change in students' overall performance during the five-year period between the two assessments.

Changes at the Grade 10 level were more dramatic: an improvement of almost eight percentage points between 1985 and 1990. The Grade 10 change items were taken from several different areas of the curriculum, but they were restricted to topics common to both the Mathematics 10 and Mathematics 10A curricula. Performance improved in every one of those areas. It is important to note that in the 1990 assessment students were permitted to use calculators, whereas in 1985 they were not. However, not all of the items were ones in which a calculator would be a significant asset, and other research has shown that access to a calculator sometimes increases, sometimes decreases, and sometimes does not affect students' success rate.

These results of the analysis of change in achievement over time are encouraging because they indicate that, at all three grade levels, the situation is either relatively stable or improving. While one might prefer to find that students' achievement was constantly increasing, such an outcome seems rather unlikely at the level of an entire educational system. It should also be noted that the change items deal with content that was part of the curriculum at both testing points,

thereby providing evidence that under the revised curriculum students are doing at least as well as students did on the same topics in the previous curriculum. The results do not tell us anything about topics which are new to the curriculum, but it seems reasonable to expect that the 1990 students would perform better than the 1985 students on material which had been taught to the current students but not to the previous students.

Conclusion

The 1990 mathematics assessment was the fourth in a series dating back to 1977. Each one has involved the collection of enormous amounts of data from students and teachers about the teaching and learning of mathematics in the schools of the province. Each assessment provides a highly detailed snapshot of what mathematics is being taught, by whom it is being taught, and the degree to which students are able to demonstrate skills and knowledge of the prescribed content. Such information is essential to rational planning for the continued improvement of the teaching and learning of mathematics in British Columbia schools.

The report of the 1985 assessment was written in anticipation of the adoption of a long-awaited, sweeping revision of the K-12 mathematics curriculum; and several of the recommendations contained in that report (Robitaille & O'Shea, 1985) addressed specific topics or pedagogical approaches which the new curriculum should encompass. That revised curriculum was adopted in 1987, and the current assessment marks the first occasion since its adoption when it has been possible to evaluate the extent to which that curriculum has been implemented and the effects that its adoption might have had on students' achievement and attitudes.

What do the results tell us? What have we learned about the state of mathematics education in the province after testing all students at three grade levels, and collecting information from their teachers? Are there areas of the curriculum in which students demonstrate particular strength? Are there others in which there is a clear need for improvement?

The *Provincial Report* is intended to provide responses to such questions. In the first section, we highlighted a number of findings of particular importance. In subsequent sections, we identified other findings that we believe accurately reflect the state of the teaching and learning of mathematics in British Columbia. Many of those results are encouraging, but others point to areas where improvement is needed. Our report is addressed to all of the major stakeholders in mathematics education in the province: teachers, principals, consultants, district-level administrators, teacher educators, the British Columbia Association of Mathematics Teachers, and the Ministry of Education. Our hope is that the report will serve as a catalyst in the continuing process of improving the teaching and learning of mathematics in our schools.

In previous mathematics assessments, we addressed specific recommendations to individual agencies—school districts, the Ministry of Education, or the Faculties of Education, for example—regarding matters where some sort of remedial action was deemed to be warranted. For this report, we decided not to make recommendations in that formal sense. Instead, we have identified our concerns throughout the report, and suggested possible directions for improvement. We hope that everyone who has a role to play in mathematics education in this province will consider our findings and take appropriate action.

Overall, the results of the 1990 assessment demonstrate that students in the schools of British Columbia are learning a great deal of mathematics and that

the levels of performance exhibited on the assessment items were, in many cases, at or above the levels established by the interpretation panels. This finding provides confirmation of the results of previous assessments of mathematics in the province, as well as from two recent international studies in which students from British Columbia have participated (Robitaille & Garden, 1989; Lapointe, Mead, Phillips, 1989).

The assessment has also led to the identification of a number of areas in which improvement is needed, and references to those have been made throughout the report. These are the areas that, in the opinion of the members of the contract team, require special attention.

In this concluding section we identify four areas or themes that we consider to be of overarching concern. Our goal is to bring these themes to the attention of major stakeholders, in the hope that the individuals and organizations involved will recognize the importance of these matters and take appropriate action to improve the current situation. The four areas are participation of women, implementation of the revised curriculum, teaching methods, and student outcomes.

Area of Concern: Participation of Women

Only 20 percent of Grade 10 mathematics teachers are women. This finding should be a matter of concern to everyone with an interest in schools, particularly when viewed in conjunction with the negative attitudes exhibited in the responses of Grade 10 girls toward continued participation in mathematics-related studies at the postsecondary level or in mathematics-related careers. There is a shortage of female teachers to serve as role models for young women who have the ability to succeed in mathematics; and, although the participation rates of boys and girls in mathematics courses at the senior secondary levels are fairly close, the differences increase dramatically in mathematics and in some of the sciences when students enter university where, among other things, future teachers acquire their academic and professional preparation.

Postsecondary educational institutions throughout the province are aware of the fact that women are seriously under-represented in the sciences generally, and many of them have adopted programs of various kinds to address the problem. The Ministry of Advanced Education and Job Training is also aware of this problem and its implications for the future of the province. Meetings have been held to discuss the extent of the problem, and to try to identify actions to be taken to reverse this trend.

We support these initiatives, but our concern is that not enough is being done, and that the problem is not being accorded the serious attention it deserves. We believe that the Ministry of Education and the Ministry of Advanced Education and Job Training should establish a task force on women in science, with a mandate to develop programs aimed at educating young women about the importance of careers in science, including teaching. The provincial universities should set up similar task forces, and the faculties of education should develop programs to attract increasing numbers of women into teaching careers in mathematics and science at the secondary school level. Schools and school districts, in cooperation with the British Columbia Teachers' Federation and the College of

Teachers, should develop incentive programs to increase the numbers of women teaching mathematics at the secondary level.

The under-representation of women in the ranks of mathematics teachers is not a new phenomenon, and the issue has been raised in a variety of settings. One of the reasons advanced as a possible explanation for that under-representation has to do with the nature of curricula in mathematics and science, and another with the pedagogical approaches which predominate in the teaching of mathematics. There is a growing school of thought which suggests that the teaching techniques most frequently employed in the teaching of mathematics—large-group lectures and individual work on assignments by students—are not highly compatible with women's ways of knowing and learning. It has also been suggested that pedagogical practices and the content of the curriculum should place less emphasis on abstraction and give increased attention to cooperative approaches to teaching and learning.

We believe that research should be carried out in the province to investigate linkages between the content of the curriculum in mathematics and science and the under-representation of women in science-related careers, including the teaching of mathematics and science. Such a research study should be the subject of a call for proposals issued to researchers and research agencies in British Columbia and elsewhere. The study should include a thorough review of the existing literature in the field, but should also include collection of pertinent data from secondary and postsecondary students in British Columbia.

Curriculum and pedagogy are two aspects of the teaching-learning process which are amenable to change, and the importance of this issue makes it imperative that solutions be sought without delay. If it is the case that the content of the mathematics curriculum and the teaching techniques which are used predominantly in mathematics classrooms are among the factors which influence young women's decisions about educational or career choices, then changes must be initiated.

Area of Concern: Implementation of the Curriculum

Initial implementation of the revised mathematics curriculum took place in 1987. Therefore, the new curriculum had been in place for at most three years when this assessment was conducted. Data from the teacher questionnaires indicate that teachers are generally supportive of the new curriculum and of the textbooks approved for use in mathematics classes. Our concern arises from the gap that seems to exist between the support in principle given to the revised curriculum and the actual implementation of that curriculum by teachers.

Teachers are professionals who are called upon daily to make decisions about what content to teach, how best to teach it, and how to adapt both content and method to the particular needs and interests of their students. For that reason, one should not be surprised to find certain differences between the intended curriculum—the curriculum as specified in the curriculum guide—and the implemented curriculum—the curriculum as taught in each classroom. However, the number of topics which teachers reported that they had not taught, and the reasons they gave for not teaching those topics, are cause for concern. In addition, information obtained from students regarding teaching practices employed

by teachers seemed to indicate that a number of important areas are either being neglected or paid insufficient attention.

Teachers reported that there was too much content in the new curriculum, and that might be a reason for not covering all of the topics in that curriculum. However, teachers also said that lack of time for teaching mathematics was not an important factor in accounting for students' lack of success in mathematics. Moreover, it seems to be the case that the amount of time teachers are devoting to the teaching of mathematics has decreased since 1985. It is difficult to make precise comparisons because the data were coded differently in 1985 than in 1990, but this finding does seem to be supported by the data. We view this as an alarming development and believe that steps should be taken to reverse the trend.

One possible explanation for this apparent decrease, especially at the elementary level, may be that teachers are devoting more class time to work on integrated themes, in which some mathematics is taught in conjunction with several other subjects in a common context. Such a trend is to be encouraged so that students come to see how mathematics is used in all aspects of life. On the other hand, the nature of mathematics requires that the curriculum be carefully structured and that topics be introduced in a well-defined sequence.

An appropriate balance between these two approaches is called for. It would be unwise to recommend that mathematics be taught in isolation from all other subjects. It would be equally unwise to think that all mathematics could be taught through integrated approaches.

Some new topics have been added to the curriculum at every grade level and others have been accorded increased emphasis in the curriculum. An example of the former is the data analysis strand which includes topics from statistics and elementary probability. An example of the latter is an increased emphasis on problem solving and teaching students how to use problem solving strategies in the solution of non-routine problems. The assessment data indicate that many of these topics are either not being taught or not being given the emphasis they deserve and require.

A possible reason for this may be that some teachers are not familiar with the new curriculum and are not making sufficient use of the curriculum guide in deciding which topics to treat and the degree of emphasis to give them. They may be relying too heavily on textbooks whose contents do not adequately reflect the topics in the curriculum guides. With all the attention that is currently being given to Year 2000 initiatives and the primary, intermediate, and graduation program documents, the prescribed curriculum guides may not be receiving the attention they deserve. Another possible explanation has to do with professional development. Many teachers were apparently unaware of any in-service opportunities in mathematics being available in their school districts, and we assume that, in at least some cases, they were probably correct in that view.

Mathematics is too important a part of the school curriculum, and the implementation of the new curriculum which has been several years in development is too important a process to be allowed to occur in anything but a structured, organized fashion. These results indicate to us a need for high quality, pro-

fessional development opportunities for teachers, with particular emphasis on those topics which are new to the curriculum and which the results of this assessment have shown to be underimplemented. All administrative levels of the school system, the faculties of education, as well as the teachers themselves and their associations, have a shared responsibility to ensure that the programs developed and adopted by the Ministry of Education are implemented as intended in the schools of the province.

Area of Concern: Teaching Methods

Students were asked to comment on the frequency with which their teachers employed a number of teaching practices. The list included whole-class instruction, quizzes and tests, seat work by individual students, teachers providing assistance to individual students, cooperative learning, and use of concrete materials. The results should be interpreted cautiously because these are data about teachers collected from students rather than from direct observation of teachers in their classrooms. However, the patterns suggested by the students' responses indicate a need for further study if not immediate action.

Research in education has shown that using concrete materials in the teaching of mathematical concepts and encouraging students to work cooperatively in small groups can result in significant improvement in students' learning and achievement. The revised curriculum guides for mathematics, on the basis of that research, encourage teachers to implement these kinds of teaching practices. Yet, the questionnaire results suggest that many teachers have not done so to any significant degree. The fact that many students express the opinion that using manipulative materials is of low importance, suggests that they believe that their teachers assign low importance to the use of concrete materials.

The use of cooperative learning groups is another pedagogical practice that seems to be used infrequently. Similarly, students' performance on the non-routine problem-solving items suggests that they may have not had much exposure to strategies for solving non-routine problems.

Area of Concern: Student Outcomes

Students in British Columbia are learning a great deal of mathematics, and the results from this assessment are encouraging overall. However, there are a number of areas where improvement is greatly to be desired, and several of these are described in the following paragraphs.

It is disconcerting to see the gradual but constant deterioration of students' attitudes toward mathematics and to the likelihood that they might pursue mathematics-related careers. The vast majority of students at all grade levels agreed with the statement that one needs to know mathematics in order to "get a good job." It is interesting to speculate about what students might have had in mind in indicating such a high degree of agreement since, on the face of it, the statement is certainly false. More disturbing, however, is the fact that almost the same proportion of students who agree with that statement say that they would not like to have a job that required them to use mathematics. These are attitudes which our society can ill afford, and everyone connected with the teaching and learning of mathematics at any level needs to be conscious of the importance of

developing positive attitudes toward the subject and the necessity of inculcating such attitudes in our students.

Students continue to have difficulty with certain topics which have traditionally had a prominent place in the curriculum. Every assessment has pointed to the difficulties students seem to have with items associated with understanding of rational number concepts and their applications. Similar difficulties are evident in the results of this assessment, and we need to search for methods of making these important ideas more accessible to students.

Geometry is another area where performance was generally below expectations. However, in this case, the level of performance seems to have more to do with the fact that some teachers are not teaching the content than with the inherent difficulty of the content. Teachers should implement the curriculum, in spirit as well as in content, and they should have access to professional development programs designed to help them do so.

Some new topics have been added to the revised curriculum, and others have been given increased emphasis. Three which deserve special mention are problem solving, estimation, and data analysis, including elementary ideas of probability. The absolute centrality of all three of these topics to the contemporary mathematics curriculum has been underscored both locally and internationally on many occasions. Yet, the assessment results show that many teachers in the province are not devoting sufficient time to them. Indeed, many teachers apparently believe that these topics are not part of the prescribed curriculum, in spite of the fact that the curriculum guide devotes a great deal of attention to them. Measures need to be taken to ensure that teachers have access to the curriculum guides as well as to textbooks and other teaching materials which will make it easier for them to include these new topics and to give them appropriate emphasis in their teaching.

References

Lapointe, A. E., Mead, N. A., & Phillips, G. W. (1989). *A World of Differences: An International Assessment of Mathematics and Science.* Princeton: Educational Testing Service.

Robitaille, D. F. & Garden, R.A. (1989). (Eds.). *The IEA Study of Mathematics II: Contexts and Outcomes of School Mathematics.* Oxford: Pergamon Press.

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Appendix A

About the Authors

David Robitaille is professor of mathematics education and head of the Department of Mathematics and Science Education at the University of British Columbia. He has been the chairperson of the Contract Teams for all four mathematics assessments in the province to date, and was the research director for curriculum for the Sullivan Royal Commission on Education. He coordinated British Columbia participation in the Second International Mathematics Study, and is serving as the International Coordinator for the Third International Mathematics and Science Study.

Thomas Schroeder is associate professor of mathematics education in the Department of Mathematics and Science Education at the University of British Columbia. He currently serves as National Council of Teachers of Mathematics (NCTM) Coordinator for the Executive of the British Columbia Association of Mathematics Teachers (BCAMT), and has been involved in evaluation projects in mathematics education in British Columbia, Alberta, and elsewhere.

Cynthia Nicol is a doctoral student in mathematics education in the Department of Mathematics and Science Education at the University of British Columbia. She is currently involved in the Third International Mathematics and Science Study centered at the University of British Columbia.

Appendix B

Place	School District #
Fernie	1
Cranbrook	2
Nelson	7
Vernon	22
Quesnel	28
Chilliwack	33
Abbotsford	34
Delta	37
Richmond	38
New Westminster	40
Burnaby	41
Maple Ridge	42
Powell River	47
Howe Sound	48
Prince Rupert	52
Peace River South	59
Saanich	63
Cowichan	65
Alberni	70
Courtenay	71
Campbell River	72
Terrace	88

Table B-1. List of pilot Districts.

Name	Place	School District #
Bob Belcher	Sooke	62
Chuck Morrison	Campbell River	72
Wendy Swonnell	Greater Victoria	61
Hélène Sullivan	Nanaimo	68
Gilles Bonenfant	Saanich	63
Marilyn Jordan	Coquitlam	43
Mark Mahovlich	Saanich	63
Grace Fraser	Surrey Curriculum Centre	36
Amy Brydon	Courtenay	71
Dana Close	Sooke	62
Cheryl MacLeod	Surrey	36
Gladys A. Bittner	Surrey	36
Alice Zilber	Surrey	36
Mignonne Wood	Burnaby	41
Ian deGroot	North Vancouver	44
Betty Milne	Cowichan	65
Sandy Dawson	Simon Fraser University	
James Sherrill	University of British Columbia	
Jim Gaskill	Ministry of Education	
Vaughan Lewis	Ministry of Education (after September 1989)	
Bill Toth (Chair)	Ministry of Education	

Table B-2. Advisory Committee.

Table B-3. Item Review Panel meeting: Richmond, B.C. Thursday, May 25, 1989. List of participants.

Grade	Place	School District #
Grade 4		
Chris Guernsey	Delta	37
Norma Dechene	Richmond	38
Alan Andrews	Vancouver	39
Bob Dall	Sunshine Coast	46
Gwen Langford	Greater Victoria	61
Bev Byron	Gulf Islands	64
Liz Matthews	Nanaimo	68
Donna Parker	Qualicum	69
Neill Dixon	Courtenay	71
Leslie Anderson	Mission	75
Grade 7		
Richard Savage	Hope	32
Jim Van Meer	Abbotsford	34
Keith Enos	Langley	35
Clare Hansen	Vancouver	39
Joanne Shutesk	Burnaby	41
Barbara Ward	Coquitlam	43
Shirley Wass	Sooke	62
Doug Alderson	Saanich	63
Bill Cleary	Cowichan	65
Allan Todd	Campbell River	72
Math 10/10A		
Pat Barrett	Surrey	36
Blair Peacock	Delta	37
Steve Taylor	Richmond	38
Louise Paulson	New Westminster	40
Phil Mendez	Burnaby	41
Steve Demos	Maple Ridge	42
Gina Thorstenson	Coquitlam	43
John Nakmoto	North Vancouver	44
Arnold Scheck	Powell River	47
Ken Floyd	Lake Cowichan	66
Barry Blair	Alberni	70
Arlene Fehr	Vancouver Island West	84
Paul Tinney	Vancouver Island North	85
Pearl Gervais	Ministry of Education	
Bill Toth	Ministry of Education	

	Place	School District #
Contract Team Member:		
Thomas Schroeder	University of British Columbia	
Gillian Clouthier	Richmond	38
Advisory Team Member:		
Cheryl MacLeod	Surrey	36
Teacher Panel:		
Jane Lindquist	Penticton	15
Stuart Culver	Keremeos	16
Dennis Hamaguchi	Vernon	22
Steve Leahy	Central Okanagan	23
Peter Marunchuck	Central Okanagan	23
Art Blackwell	Kamloops	24
John McLellan	Kamloops	24
Catherine Davies	Lillooet	29
Margaret Todd	South Cariboo	30
Karen Andreassen	Shuswap	89
Bill Toth	Ministry of Education	

Table B-4. Item Review Panel Meeting: Delta Lakeside, Penticton, B.C. Wednesday, May 31, 1989. List of participants. Grade 4.

	Place	School District #
Contract Team Member:		
Les Dukowski	Langley	35
Advisory Team Member:		
Grace Fraser	Surrey	36
Teacher Panel:		
Rob Gallamore	Quesnel	28
Brent Steele	Bulkley Valley	54
Jan Bradley	Burns Lake	55
Dan Costain	Nechako	56
Barry Huhn	Nechako	56
Norm Williams	Prince George	57
Dave King	Prince George	57
Donna Dojack	Prince George	57
Ray Asai	Peace River North	60
Brian Stevens	Terrace	88
Bill Toth	Ministry of Education	

Table B-5. Item Review Panel Meeting: Inn of the North, Prince George, B.C. Friday, June 2, 1989. List of participants, Grade 7.

Table B-6. Item Review Panel Meeting: Inn of the South, Cranbrook, B.C. Monday, May 29, 1989. List of participants, Math 10/10A.

	Place	School District #
Contract Team:		
Alan Taylor	Coquitlam	
Bill Kokoskin	North Vancouver	
Advisory Panel Member:		
Chuck Morrison	Campbell River	72
Teacher Panel		
Richard Pinotti	Fernie	1
Jim Thompson	Cranbrook	2
Grant Stewart	Cranbrook	2
Marvin Smith	Kimberley	3
Tony Kraayvanger	Windermere	4
Ed Devries	Nelson	7
Peter Young	Castlegar	9
Gary Davidson	Arrow Lakes	10
Art Benzer	Trail	11
Keith Farnworth	Grand Forks	12
Emilie Belak	Grand Forks	12
Jeff Moss	Golden	18
Gary Lewis	Revelstoke	19
Gerry Schiavon	Creston-Kaslo	86
Bill Toth	Ministry of Education	

Table B-7. French Translation Review Panel.

Name	Place	School District #
Vaughan Lewis (Chair)	Ministry of Education	
Catherine Eberlé	Vancouver	39
Lilianne Doucet	Ministry of Education	
Gloria Gietz	Ministry of Education	
Marie-France Castex	Ministry of Education	
Stephen Long	Ministry of Education	
Régine Pasquier	Contractor	
Marilyn Jordan	Coquitlam	43
Gilles Bonenfant	Saanich	63
Edèle Bonnaig	Greater Victoria	61
Hélène Sullivan	Nanaimo	68

Name	Place	School District #
Bob Belcher	Sooke	62
Amy Brydon	Courtenay	71
Dana Close	Sooke	62
Ivan Johnson	Burnaby	41
Marilyn Jordan	Coquitlam	43
Jack Kinakin	Castlegar	09
Carryl Koe	Mission	75
Bill Toth	Ministry of Education	

Table B-8. Marking Chairs, July, 1990.

Name	Place	School District #
Sandy Adams	Fort Nelson	81
John Briggs	Vancouver Island North	85
Rennie Brown	Nisga'a	92
Cathy Bulger	Courtenay	71
Ron Burger	Bulkley Valley	54
Glen Carter	Fernie	1
Neil Caves	Shuswap	89
Val Charison	Central Okanagan	23
Mauri Clemons - Braun	Vancouver	39
Kate Dudley	Coquitlam	43
Carol Edy	North Vancouver	96
Keith Enns	Langley	35
Bonnie Field	Delta	37
Max Geisthardt	Cowichan	65
Bryan Hansen	Peace River North	60
Glen Kinder	Creston–Kaslo	86
Janet Madhok	Nechako	56
Parmjit Parmar	Maple Ridge	42
Dennis Richards	North Vancouver	44
Judy Robinson	Abbotsford	34
Tom Shaw	Vancouver Island West	84
Elaine Sturgeon	Penticton	15
Carla Taylor	New Westminster	40
Mirella Wessel	Kamloops	24

Table B-9. Problem Solving Markers, Grade 4. July, 1990.

Table B-10. Problem Solving markers, Grade 7. July, 1990.

Name	Place	School District #
Bruce Anderson	Nechako	56
Robert Bardot	Central Okanagan	23
Lynn Broman	Vancouver	39
Ray Brunelle	North Vancouver	44
Bob Campbell	Greater Victoria	61
Judy Devereaux	Coquitlam	43
Susan Diewert	Richmond	38
Katalin Duval	Langley	35
Chris Hogarth	Campbell River	72
Dan Jorgensen	Abbotsford	34
Arthur MacNeil	Trail	11
Reusse Manfred	North Thompson	26
Margaret McDonough	West Vancouver	45
Donna McLean	Penticton	15
Al McRitchie	Surrey	36
Terry Molnar	Cranbrook	2
Bruce Murray	Kamloops	24
Bob Peacock	Terrace	88
Bill Rudyk	Revelstoke	19
Nevenka Salapura	Campbell River	72
Charles Schellinck	Golden	18
Philip Sorensen	Alberni	70
Dean Watson	Cariboo-Chilcotin	27
Norman Williams	Prince George	57

Table B-11. Problem Solving Markers, Grade 10. July, 1990.

Name	Place	School District #
Jo-Anne Aura	Peace River South	59
Barry Blair	Alberni	70
Gordon Burleson	Vancouver Island North	85
Dan Dobrinsky	Quesnel	28
Keith Farnworth	Grand Forks	12
Bonnie Fuller	Prince George	57
Lori Giacometti	Cariboo-Chilcotin	27
George Hawkins	Kettle Valley	13
Peter Isman	Vernon	22
Ameer Khan	Qualicum	69
Ed Klette	Chilliwack	33
John Livam	South Cariboo	30
Tony McCrory	Nanaimo	68
Jeff Moss	Golden	18
Wendy Mundie	Bulkley Valley	54
Frank Perehudoff	Castlegar	9
Allan Rasnysseb	Merritt	31
Keith Robertson	Cowichan	65
Naida Schulz	Central Coast	49
Steve Tarasoff	Nelson	7
Stella Tossell	Vancouver	39
Rich Tschritter	Greater Victoria	61
Andy Wong	Gulf Islands	64
Pamela Yates	Burnaby	41

Name	Place	School District #
Gaétan Dupont	Surrey	36
Lain Martin	Central Okanagan	23
Monica Rott	Cariboo-Chilcotin	27
Judy Toy	Coquitlam	43
Marc Tremblay	West Vancouver	45

Table B-12. Problem Solving Markers - French. July, 1990

Name	Place	Status
Anne Boyd	Campbell River (# 72)	Teacher
Marilyn Bueckert	Mission (# 75)	Teacher
Joy Davidson	West Vancouver	Parent
Darryl Guza	Penticton (# 15)	Teacher
Dennis Hamaguchi	Vernon (# 22)	Chair
Dan Hudy	Cranbrook (# 2)	Teacher
Wendy Lee	North Vancouver (# 44)	Teacher
Peter Lock	Powell River (# 47)	Teacher
Colleen Lougheed	Victoria	University
Judy McLeod	Delta (# 37)	Teacher
Linda Moir	Langley (# 35)	Trustee
Susan Richardson	Kamloops (# 24)	Teacher
Susan Vlismas	Coquitlam (# 43)	Teacher
Debbie Watteyne	Lillooet (# 29)	Teacher

Table B-13. Interpretation Panel, Grade 4. July, 1990.

Name	Place	Status
Dave Bradley	Richmond (# 38)	Teacher
Steve Cairns	Burnaby (# 41)	Teacher
Kim Doerksen	Chetwynd	Parent
Colin Green	Greater Victoria (# 61)	Teacher
June McClure	Nechako (# 56)	Teacher
Freda O'Sullivan	Sooke (# 62)	Chair
Valerie Peterson	Nelson (# 7)	Teacher
Jerald Reed	Maple Ridge (# 42)	Teacher
Jim Sherrill	U.B.C.	University
Lynn Skriyc	Port McNeill	Trustee
Karen Susheski	Surrey (# 36)	Teacher
Dean Watson	Cariboo-Chilcotin (# 27)	Teacher
Douglas Wiebe	Central Okanagan (# 23)	Teacher
Norman Williams	Prince George (# 57)	Teacher

Table B-14. Interpretation Panel, Grade 7, July, 1990.

Table B-15. Interpretation Panel, Math 10. July, 1990.

Name	Place	Status
Hedwig Braunwarth	Cowichan (# 65)	Teacher
Donald Caskey	Grand Forks (# 12)	Teacher
Catherine Eberlé	Vancouver (# 39)	Chair
David Ellis	Vancouver (# 39)	Teacher
Ken Harper	West Vancouver	University
John MacMaster	Trail (# 11)	Teacher
Mark Mahovlich	Saanich (# 63)	Co-ordinator
Peggy Matheson	Burnaby (# 41)	Teacher
Rose Paszty	Castlegar	Parent
Lorne Peter	Vernon (# 22)	Teacher
Heidi Robinson	Abbotsford (# 34)	Teacher
John Runke	Nanaimo (# 68)	Teacher
Sally Sadler	Richmond (# 38)	Teacher
Evelyn Voykin	Castlegar (# 9)	Trustee
Alice Zilber	Surrey (# 36)	Chair

Table B-16. Interpretation Panel, French. July, 1990.

Name	Place	Grade	Status
John Breland	Central Okanagan (# 23)	4	Teacher
Gaétan Dupont	Surrey (# 36)	10	Teacher
Simone Dupont	Coquitlam (# 43)	4	Teacher
Catherine Eberlé	Vancouver (# 39)	10	Chair
Claudine Le Goff	Greater Victoria (# 61)	7	Teacher
Vaughan Lewis	Ministry of Education		Co-ordinator

Table B-17. Interpretation Panel, Math 10A. September, 1990.

Name	Place	School District #
Barry Blair	Alberni	70
Mark Mahovlich	Saanich	63
Peggy Matheson	Burnaby	41
Tom Poulton	Delta	37
Keith Robertson	Cowichan	65
John Runke	Nanaimo	68
Stella Tossell	Vancouver	39

Name	Place	School District #
Bob Belcher	Sooke	62
Lynn Broman	Vancouver	39
Bob Campbell	Greater Victoria	61
Judy Devereaux	Coquitlam	43
Kate Dudley	Coquitlam	43
Keith Enns	Langley	35
Bonnie Field	Delta	37
Max Geisthardt	Cowichan	65
Dan Jorgensen	Abbotsford	34
Ed Klette	Chilliwack	33
Carryl Koe	Delta	37
Tony McCrory	Nanaimo	68
Margaret McDonough	North Vancouver	44
Dennis Richards	North Vancouver	44
Keith Robertson	Cowichan	65
Carla Taylor	New Westminster	40
Stella Tossell	Vancouver	39
Richard Tschritter	Greater Victoria	61
Pamela Yates	Burnaby	41
Amy Brydon	Ministry of Education	
Dana Close	Ministry of Education	
Bill Toth	Ministry of Education	

Table B-18. Interpetation Panel. Open-Ended Problems, Q Forms. September, 1990.

Vern A. Burkhardt
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